# MATP®WER User's Manual 

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## Contents

1 Introduction ..... 10
1.1 Background ..... 10
1.2 License and Terms of Use ..... 11
1.3 Citing Matpower ..... 12
1.4 Matpower Development ..... 12
2 Getting Started ..... 13
2.1 System Requirements ..... 13
2.2 Getting Matpower ..... 14
2.2.1 Versioned Releases ..... 14
2.2.2 Current Development Version ..... 14
2.3 Installation ..... 16
2.4 Running a Simulation ..... 18
2.4.1 Preparing Case Input Data ..... 18
2.4.2 Solving the Case ..... 18
2.4.3 Accessing the Results ..... 19
2.4.4 Setting Options ..... 20
2.5 Documentation ..... 20
3 Modeling ..... 23
3.1 Data Formats ..... 23
3.2 Branches ..... 23
3.3 Generators ..... 25
3.4 Loads ..... 25
3.5 Shunt Elements ..... 26
3.6 Network Equations ..... 26
3.7 DC Modeling ..... 27
4 Power Flow ..... 30
4.1 AC Power Flow ..... 30
4.2 DC Power Flow ..... 32
4.3 Distribution Power Flow ..... 32
4.3.1 Radial Power Flow ..... 33
4.3.2 Current Summation Method ..... 34
4.3.3 Power Summation Method ..... 35
4.3.4 Admittance Summation Method ..... 35
4.3.5 Handling PV Buses ..... 37
4.4 runpf ..... 38
4.5 Linear Shift Factors ..... 39
5 Continuation Power Flow ..... 43
5.1 Parameterization ..... 44
5.2 Predictor ..... 44
5.3 Corrector ..... 45
5.4 Step Length Control ..... 45
5.5 Event Detection and Location ..... 46
5.6 runcpf ..... 47
5.6.1 CPF Callback Functions ..... 50
5.6.2 CPF Example ..... 53
6 Optimal Power Flow ..... 56
6.1 Standard AC OPF ..... 56
6.1.1 Cartesian vs. Polar Coordinates for Voltage ..... 57
6.1.2 Current vs. Power for Nodal Balance Constraints ..... 58
6.2 Standard DC OPF ..... 59
6.3 Extended OPF Formulation ..... 60
6.3.1 User-defined Variables ..... 61
6.3.2 User-defined Constraints ..... 61
6.3.3 User-defined Costs ..... 62
6.4 Standard Extensions ..... 64
6.4.1 Piecewise Linear Costs ..... 64
6.4.2 Dispatchable Loads ..... 66
6.4.3 Generator Capability Curves ..... 68
6.4.4 Branch Angle Difference Limits ..... 69
6.5 Solvers ..... 69
6.6 runopf ..... 70
7 Extending the OPF ..... 76
7.1 Direct Specification ..... 76
7.1.1 User-defined Variables ..... 76
7.1.2 User-defined Constraints ..... 76
7.1.3 User-defined Costs ..... 78
7.1.4 Additional Comments ..... 79
7.2 Callback Functions ..... 79
7.2.1 User-defined Variables ..... 80
7.2.2 User-defined Costs ..... 80
7.2.3 User-defined Constraints ..... 81
7.3 Callback Stages and Example ..... 82
7.3.1 ext2int Callback ..... 83
7.3.2 formulation Callback ..... 84
7.3.3 int2ext Callback ..... 88
7.3.4 printpf Callback ..... 91
7.3.5 savecase Callback ..... 93
7.4 Registering the Callbacks ..... 95
7.5 Summary ..... 97
7.6 Example Extensions ..... 97
7.6.1 Fixed Zonal Reserves ..... 97
7.6.2 Interface Flow Limits ..... 99
7.6.3 DC Transmission Lines ..... 100
7.6.4 OPF Soft Limits ..... 103
8 Unit De-commitment Algorithm ..... 110
9 Miscellaneous Matpower Functions ..... 112
9.1 Input/Output Functions ..... 112
9.1.1 loadcase ..... 112
9.1.2 savecase ..... 112
9.1.3 cdf2mpc ..... 113
9.1.4 psse2mpc ..... 113
9.1.5 save2psse ..... 114
9.2 System Information ..... 114
9.2.1 case_info ..... 114
9.2.2 compare_case ..... 114
9.2.3 find_islands ..... 115
9.2.4 get_losses ..... 115
9.2.5 margcost ..... 116
9.2.6 isload ..... 116
9.2.7 loadshed ..... 116
9.2.8 printpf ..... 116
9.2.9 total_load ..... 117
9.2.10 totcost ..... 117
9.3 Modifying a Case ..... 117
9.3.1 extract_islands ..... 117
9.3.2 load2disp ..... 118
9.3.3 modcost ..... 118
9.3.4 scale_load ..... 118
9.3.5 apply_changes ..... 119
9.3.6 savechgtab ..... 122
9.4 Conversion between External and Internal Numbering ..... 123
9.4.1 ext2int, int2ext ..... 123
9.4.2 e2i_data, i2e_data ..... 123
9.4.3 e2i_field, i2e_field ..... 124
9.5 Forming Standard Power Systems Matrices ..... 125
9.5.1 makeB ..... 125
9.5.2 makeBdc ..... 125
9.5.3 makeJac ..... 125
9.5.4 makeLODF ..... 126
9.5.5 makePTDF ..... 126
9.5.6 makeYbus ..... 126
9.6 Miscellaneous ..... 127
9.6.1 define_constants ..... 127
9.6.2 feval_w_path ..... 127
9.6.3 have_fcn ..... 127
9.6.4 mpopt2qpopt ..... 128
9.6.5 mpver ..... 129
9.6.6 nested_struct_copy ..... 129
10 Acknowledgments ..... 130
Appendix A MIPS - Matpower Interior Point Solver ..... 131
A. 1 Example 1 ..... 133
A. 2 Example 2 ..... 135
A. 3 Quadratic Programming Solver ..... 137
A. 4 Primal-Dual Interior Point Algorithm ..... 138
A.4.1 Notation ..... 138
A.4.2 Problem Formulation and Lagrangian ..... 139
A.4.3 First Order Optimality Conditions ..... 140
A.4.4 Newton Step ..... 141
Appendix B Data File Format ..... 144
Appendix C Matpower Options ..... 150
C. 1 Mapping of Old-Style Options to New-Style Options ..... 166
Appendix D Matpower Files and Functions ..... 170
D. 1 Directory Layout and Documentation Files ..... 170
D. 2 Matpower Functions ..... 172
D. 3 Example Matpower Cases ..... 184
D. 4 Automated Test Suite ..... 187
Appendix E Extras Directory ..... 191
Appendix F "Smart Market" Code ..... 193
F. 1 Handling Supply Shortfall ..... 195
F. 2 Example ..... 195
F. 3 Smartmarket Files and Functions ..... 200
Appendix G Optional Packages ..... 201
G. 1 BPMPD_MEX - MEX interface for BPMPD ..... 201
G. 2 CLP - COIN-OR Linear Programming ..... 201
G. 3 CPLEX - High-performance LP and QP Solvers ..... 202
G. 4 GLPK - GNU Linear Programming Kit ..... 203
G. 5 Gurobi - High-performance LP and QP Solvers ..... 203
G. 6 Ipopt - Interior Point Optimizer ..... 204
G. 7 KNITRO - Non-Linear Programming Solver ..... 205
G. 8 MINOPF - AC OPF Solver Based on MINOS ..... 206
G. 9 MOSEK - High-performance LP and QP Solvers ..... 206
G. 10 Optimization Toolbox - LP, QP, NLP and MILP Solvers ..... 207
G. 11 PARDISO - Parallel Sparse Direct and Multi-Recursive Iterative Lin- ear Solvers ..... 207
G. 12 SDP_PF - Applications of a Semidefinite Programming Relaxation of the Power Flow Equations ..... 208
G. 13 TSPOPF - Three AC OPF Solvers by H. Wang ..... 208
Appendix H Release History ..... 210
H. 1 Pre 1.0 - released Jun 25, 1997 ..... 210
H. 2 Version 1.0 - released Sep 17, 1997 ..... 210
H. 3 Version 1.0.1 - released Sep 19, 1997 ..... 210
H. 4 Version 2.0 - released Dec 24, 1997 ..... 211
H. 5 Version 3.0 - released Feb 14, 2005 ..... 212
H. 6 Version 3.2 - released Sep 21, 2007 ..... 213
H. 7 Version 4.0 - released Feb 7, 2011 ..... 215
H. 8 Version 4.1 - released Dec 14, 2011 ..... 218
H. 9 Version 5.0 - released Dec 17, 2014 ..... 219
H. 10 Version 5.1 - released Mar 20, 2015 ..... 224
H. 11 Version 6.0 - released Dec 16, 2016 ..... 227
H. 12 Version 7.0 - beta 1 released released Oct 31, 2018 ..... 232
References ..... 240

## List of Figures

3-1 Branch Model ..... 24
4-1 Oriented Ordering ..... 33
4-2 Branch Representation: branch $k$ between buses $i$ (sending) and $k$ (receiving) and load demand and shunt admittances at both buses ..... 34
5-1 Nose Curve of Voltage Magnitude at Bus 9 ..... 53
6-1 Relationship of $w_{i}$ to $r_{i}$ for $d_{i}=1$ (linear option) ..... 63
6-2 Relationship of $w_{i}$ to $r_{i}$ for $d_{i}=2$ (quadratic option) ..... 64
6-3 Constrained Cost Variable ..... 65
6-4 Marginal Benefit or Bid Function ..... 66
6-5 Total Cost Function for Negative Injection ..... 67
6-6 Generator $P-Q$ Capability Curve ..... 68
7-1 Adding Constraints Across Subsets of Variables ..... 87
7-2 DC Line Model ..... 101
7-3 Equivalent "Dummy" Generators ..... 101
7-4 Feasible Region for Branch Flow Violation Constraints ..... 104
List of Tables
4-1 Power Flow Results ..... 39
4-2 Power Flow Options ..... 40
4-3 Power Flow Output Options ..... 41
5-1 Continuation Power Flow Results ..... 48
5-2 Continuation Power Flow Options ..... 49
5-3 Continuation Power Flow Callback Input Arguments ..... 51
5-4 Continuation Power Flow Callback Output Arguments ..... 52
5-5 Continuation Power Flow State ..... 52
6-1 Optimal Power Flow Results ..... 71
6-2 Optimal Power Flow Solver Options ..... 73
6-3 Other OPF Options ..... 74
6-4 OPF Output Options ..... 75
7-1 User-defined Nonlinear Constraint Specification ..... 78
7-2 Names Used by Implementation of OPF with Reserves ..... 83
7-3 Results for User-Defined Variables, Constraints and Costs ..... 89
7-4 Callback Functions ..... 97
7-5 Input Data Structures for Fixed Zonal Reserves ..... 98
7-6 Output Data Structures for Fixed Zonal Reserves ..... 98
7-7 Input Data Structures for Interface Flow Limits ..... 99
7-8 Output Data Structures for Interface Flow Limits ..... 100
7-9 Soft Limit Formulation ..... 105
7-10 Input Data Structures for OPF Soft Limits ..... 106
7-11 Default Soft Limit Values ..... 107
7-12 Possible Hard-Limit Modifications ..... 107
7-13 Output Data Structures for OPF Soft Limits ..... 108
9-1 Columns of chgtab ..... 120
9-2 Values for CT_TABLE Column ..... 120
9-3 Values for CT_CHGTYPE Column ..... 121
9-4 Values for CT_COL Column ..... 121
A-1 Input Arguments for mips ..... 132
A-2 Output Arguments for mips ..... 133
A-3 Options for mips ..... 134
B-1 Bus Data (mpc.bus) ..... 145
B-2 Generator Data (mpc.gen) ..... 146
B-3 Branch Data (mpc.branch) ..... 147
B-4 Generator Cost Data (mpc.gencost) ..... 148
B-5 DC Line Data (mpc.dcline) ..... 149
C-1 Top-Level Options ..... 152
C-2 Power Flow Options ..... 153
C-3 Continuation Power Flow Options ..... 154
C-4 OPF Solver Options ..... 155
C-5 General OPF Options ..... 156
C-6 Power Flow and OPF Output Options ..... 157
C-7 OPF Options for MIPS ..... 158
C-8 OPF Options for CLP ..... 158
C-9 OPF Options for CPLEX ..... 159
C-10 OPF Options for fmincon ..... 160
C-11 OPF Options for GLPK ..... 160
C-12 OPF Options for Gurobi ..... 161
C-13 OPF Options for Ipopt ..... 161
C-14 OPF Options for KNITRO ..... 162
C-15 OPF Options for MINOPF ..... 163
C-16 OPF Options for MOSEK ..... 164
C-17 OPF Options for PDIPM ..... 165
C-18 OPF Options for TRALM ..... 165
C-19 Old-Style to New-Style Option Mapping ..... 166
D-1 Matpower Directory Layout and Documentation Files ..... 171
D-2 Top-Level Simulation Functions ..... 172
D-3 Input/Output Functions ..... 172
D-4 Data Conversion Functions ..... 173
D-5 Power Flow Functions ..... 173
D-6 Continuation Power Flow Functions ..... 174
D-7 OPF and Wrapper Functions ..... 175
D-8 OPF Model Objects ..... 176
D-9 Deprecated @opt_model Methods ..... 177
D-10 OPF Solver Functions ..... 177
D-11 Other OPF Functions ..... 178
D-12 OPF User Callback Functions ..... 179
D-13 Power Flow Derivative Functions ..... 179
D-14 NLP, LP \& QP Solver Functions ..... 180
D-15 Matrix Building Functions ..... 181
D-16 Utility Functions ..... 182
D-17 Other Functions ..... 183
D-18 Small Transmission System Test Cases ..... 184
D-19 Small Radial Distribution System Test Cases ..... 184
D-20 ACTIV Synthetic Grid Test Cases ..... 185
D-21 Polish System Test Cases ..... 185
D-22 PEGASE European System Test Cases ..... 186
D-23 RTE French System Test Cases ..... 186
D-24 Automated Test Functions from MP-Test ..... 187
D-25 MIPS Tests ..... 187
D-26 Test Data ..... 188
D-27 Miscellaneous Matpower Tests ..... 189
D-28 Matpower OPF Tests ..... 190
F-1 Auction Types ..... 194
F-2 Generator Offers ..... 196
F-3 Load Bids ..... 196
F-4 Generator Sales ..... 199
F-5 Load Purchases ..... 199
F-6 Smartmarket Files and Functions ..... 200

## MATP@WER

## 1 Introduction

### 1.1 Background

MATPOWER [1] is a package of MATLAB ${ }^{\circledR}$ M-files for solving power flow and optimal power flow problems. It is intended as a simulation tool for researchers and educators that is easy to use and modify. Matpower is designed to give the best performance possible while keeping the code simple to understand and modify. The Matpower website can be found at:

```
http://www.pserc.cornell.edu/matpower/
```

Matpower was initially developed by Ray D. Zimmerman, Carlos E. MurilloSánchez and Deqiang Gan of PSERC ${ }^{1}$ at Cornell University under the direction of Robert J. Thomas. The initial need for Matlab-based power flow and optimal power flow code was born out of the computational requirements of the PowerWeb project ${ }^{2}$. Many others have contributed to MATPOWER over the years and it continues to be developed and maintained under the direction of Ray Zimmerman.

Beginning with version 6, MATPOWER includes a framework for solving generalized steady-state electric power scheduling problems. This framework is known as MOST, for Matpower Optimal Scheduling Tool [2, 3].

MOST can be used to solve problems as simple as a deterministic, single period economic dispatch problem with no transmission constraints or as complex as a stochastic, security-constrained, combined unit-commitment and multiperiod optimal power flow problem with locational contingency and load-following reserves, ramping costs and constraints, deferrable demands, lossy storage resources and uncertain renewable generation.

MOST is documented separately from the main MATPOWER package in its own manual, the MOST User's Manual.

[^0]
### 1.2 License and Terms of Use

Beginning with version 5.1, the code in Matpower is distributed under the 3-clause BSD license ${ }^{3}$ [5]. The full text of the license can be found in the LICENSE file at the top level of the distribution or at http://www.pserc.cornell.edu/matpower/ LICENSE.txt and reads as follows.

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[^1]Please note that the Matpower case files distributed with Matpower are not covered by the BSD license. In most cases, the data has either been included with permission or has been converted from data available from a public source.

### 1.3 Citing Matpower

While not required by the terms of the license, we do request that publications derived from the use of MATPOWER explicitly acknowledge that fact by citing reference [1].

R. D. Zimmerman, C. E. Murillo-Sánchez, and R. J. Thomas, "Matpower: SteadyState Operations, Planning and Analysis Tools for Power Systems Research and Education," Power Systems, IEEE Transactions on, vol. 26, no. 1, pp. 12-19, Feb. 2011. DOI: 10.1109/TPWRS.2010.2051168

Similarly, we request that publications derived from the use of MOST explicitly acknowledge that fact by citing both the main MATPOWER reference above and reference [2]:

> C. E. Murillo-Sánchez, R. D. Zimmerman, C. L. Anderson, and R. J. Thomas, "Secure Planning and Operations of Systems with Stochastic Sources, Energy Storage and Active Demand," Smart Grid, IEEE Transactions on, vol. 4, no. 4, pp. 2220-2229, Dec. 2013.
> DOI: $10.1109 /$ TSG. 2013.2281001

### 1.4 Matpower Development

Following the release of Matpower 6.0, the Matpower project moved to an open development paradigm, hosted on the Matpower GitHub project page:
https://github.com/MATPOWER/matpower

The Matpower GitHub project hosts the public Git code repository as well as a public issue tracker for handling bug reports, patches, and other issues and contributions. There are separate GitHub hosted repositories and issue trackers for Matpower, MOST, MIPS and the testing framework used by all of them, MP-Test, all available from https://github.com/MATPOWER/.

## 2 Getting Started

### 2.1 System Requirements

To use Matpower 7.0b1 you will need:

- Matlab ${ }^{\circledR}$ version 7.3 (R2006b) or later ${ }^{4}$, or
- GNU Octave version 4 or later ${ }^{5}$

See Section 2.1 in the MOST User's Manual for any requirements specific to MOST.

For the hardware requirements, please refer to the system requirements for the version of Matlab ${ }^{6}$ or Octave that you are using. If the Matlab Optimization Toolbox is installed as well, Matpower enables an option to use it to solve optimal power flow problems.

In this manual, references to Matlab usually apply to Octave as well. At the time of writing, none of the optional MEX-based Matpower packages have been built for Octave, but Octave does typically include GLPK.

[^2]
### 2.2 Getting Matpower

You can either download an official versioned release or you can obtain the current development version, which we also attempt to keep stable enough for everyday use. The development version includes new features and bug fixes added since the last versioned release.

### 2.2.1 Versioned Releases

- Download the ZIP file of the latest official versioned release from the MatPOWER website ${ }^{7}$.
- Go to the Matpower website.
- Click the Download Now button.


### 2.2.2 Current Development Version

There are also two options for obtaining the most recent development version of Matpower from the master branch on GitHub.

1. Clone the Matpower repository from GitHub.

- From the command line:
- git clone https://github.com/MATPOWER/matpower.git
- Or, from the Matpower GitHub repository page ${ }^{8}$ :
- Click the green Clone or download button, then Open in Desktop.

Use this option if you want to be able to easily update to the current development release, with the latest bug fixes and new features, using a simple git pull command, or if you want to help with testing or development. This requires that you have a Git client ${ }^{9}$ (GUI or command-line) installed.
2. Download a zip file of the Matpower repository from GitHub.

- Go to the Matpower GitHub repository page.

[^3]- Click the green Clone or download button, then Download ZIP.

Use this option if you need features or fixes introduced since the latest versioned release, but you do not have access to or are not ready to begin using Git (but don't be afraid to give Git a try). ${ }^{10}$

See CONTRIBUTING.md for information on how to get a local copy of your own MATPOWER fork, if you are interesting in contributing your own code or modifications.

[^4]
### 2.3 Installation

Installation and use of MATPOWER requires familiarity with the basic operation of Matlab or Octave. Make sure you follow the installation instructions for the version of Matpower you are installing. The process was simplified with an install script following version 6.0.

Step 1: Get a copy of Matpower as described above.
Clone the repository or download and extract the ZIP file of the MatPOWER distribution and place the resulting directory in the location of your choice and call it anything you like. ${ }^{11}$ We will use <MATPOWER> as a placeholder to denote the path to this directory (the one containing install matpower.m). The files in $<M A T P O W E R>$ should not need to be modified, so it is recommended that they be kept separate from your own code.

## Step 2: Run the installer.

Open Matlab or Octave and change to the $<M A T P O W E R>$ directory.
Run the installer and follow the directions to add the required directories to your Matlab or Octave path, by typing:

```
install_matpower
```

Step 3: That's it. There is no step 3.
But, if you chose not to have the installer run the test suite for you in step 2, you can run it now to verify that MATPOWER is installed and functioning properly, by typing: ${ }^{12}$

```
test_matpower
```

The result should resemble the following, possibly including extra tests, depending on the availablility of optional packages, solvers and extras.

[^5]```
>> test_matpower
t_test_fcns...............ok
t_nested_struct_copy.....ok
t_feval_w_path............ok
t_mpoption..................ok
t_loadcase...... . . . . . . . . . ok
t_ext2int2ext. . . . . . . . . . .ok
t_jacobian.....................
t_hessian...................ok
t_margcost. . . . . . . . . . . . . .ok
t_totcost...................ok
t_modcost..........................
t_hasPQcap..................ok
t_mplinsolve............ok (6 of 44 skipped)
t_mips.......................ok
t_mips_pardiso..........ok (60 of 60 skipped)
t_qps_matpower..........ok (288 of 360 skipped)
t_miqps_matpower........ok (240 of 240 skipped)
t_pf. . . . . . . . . . . . . . . . . . . ok
t_pf_radial.................ok
t_cpf..........................ok
t_islands........................
t_opf_model................ok
t_opf_model_legacy.......ok
t_opf_default. . . . . . . . . . .ok
t_opf_mips...............ok (278 of 1296 skipped)
t_opf_dc_mips..............ok
t_opf_dc_mips_sc. . . . . . . .ok
t_opf_userfcns.............ok
t_opf_softlims............ok
t_runopf_w_res. . . . . . . . . . ok
t_dcline....................ok
t_get_losses. . . . . . . . . . . .ok
t_load2disp................ok
t_makePTDF. . . . . . . . . . . . . . ok
t_makeLODF. . . . . . . . . . . . . . ok
t_printpf . . . . . . . . . . . . . . .ok
t_vdep_load......................
t_total_load...............ok
t_scale_load...............ok
t_apply_changes. . . . . . . . . ok
t_psse............................
t_off2case.....................
t_auction_mips............ok
t_runmarket................ok
All tests successful (7961 passed, }872\mathrm{ skipped of 8833)
Elapsed time 29.39 seconds.
```


### 2.4 Running a Simulation

The primary functionality of Matpower is to solve power flow and optimal power flow (OPF) problems. This involves (1) preparing the input data defining the all of the relevant power system parameters, (2) invoking the function to run the simulation and (3) viewing and accessing the results that are printed to the screen and/or saved in output data structures or files.

### 2.4.1 Preparing Case Input Data

The input data for the case to be simulated are specified in a set of data matrices packaged as the fields of a MATLAB struct, referred to as a "MATPOWER case" struct and conventionally denoted by the variable mpc. This struct is typically defined in a case file, either a function M-file whose return value is the mpc struct or a MAT-file that defines a variable named mpc when loaded ${ }^{13}$. The main simulation routines, whose names begin with run (e.g. runpf, runopf), accept either a file name or a Matpower case struct as an input.

Use loadcase to load the data from a case file into a struct if you want to make modifications to the data before passing it to the simulation.

```
>> mpc = loadcase(casefilename);
```

See also savecase for writing a MATPOWER case struct to a case file.
The structure of the Matpower case data is described a bit further in Section 3.1 and the full details are documented in Appendix B and can be accessed at any time via the command help caseformat. The MATPOWER distribution also includes many example case files listed in Table D-18.

### 2.4.2 Solving the Case

The solver is invoked by calling one of the main simulation functions, such as runpf or runopf, passing in a case file name or a case struct as the first argument. For example, to run a simple Newton power flow with default options on the 9-bus system defined in case9.m, at the Matlab prompt, type:

```
>> runpf('case9');
```

[^6]If, on the other hand, you wanted to load the 30-bus system data from case30.m, increase its real power demand at bus 2 to 30 MW , then run an AC optimal power flow with default options, this could be accomplished as follows:

```
>> define_constants;
>> mpc = loadcase('case30');
>> mpc.bus(2, PD) = 30;
>> runopf(mpc);
```

The define_constants in the first line is simply a convenience script that defines a number of variables to serve as named column indices for the data matrices. In this example, it allows us to access the "real power demand" column of the bus matrix using the name PD without having to remember that it is the $3^{\text {rd }}$ column.

Other top-level simulation functions are available for running DC versions of power flow and OPF, for running an OPF with the option for Matpower to shut down (decommit) expensive generators, etc. These functions are listed in Table D-2 in Appendix D.

### 2.4.3 Accessing the Results

By default, the results of the simulation are pretty-printed to the screen, displaying a system summary, bus data, branch data and, for the OPF, binding constraint information. The bus data includes the voltage, angle and total generation and load at each bus. It also includes nodal prices in the case of the OPF. The branch data shows the flows and losses in each branch. These pretty-printed results can be saved to a file by providing a filename as the optional $3^{\text {rd }}$ argument to the simulation function.

The solution is also stored in a results struct available as an optional return value from the simulation functions. This results struct is a superset of the Matpower case struct mpc, with additional columns added to some of the existing data fields and additional fields. The following example shows how simple it is, after running a DC OPF on the 118-bus system in case118.m, to access the final objective function value, the real power output of generator 6 and the power flow in branch 51 .

```
>> define_constants;
>> results = rundcopf('case118');
>> final_objective = results.f;
>> gen6_output = results.gen(6, PG);
>> branch51_flow = results.branch(51, PF);
```

Full documentation for the content of the results struct can be found in Sections 4.4 and 6.6.

### 2.4.4 Setting Options

MATPOWER has many options for selecting among the available solution algorithms, controlling the behavior of the algorithms and determining the details of the prettyprinted output. These options are passed to the simulation routines as a MATPOWER options struct. The fields of the struct have names that can be used to set the corresponding value via the mpoption function. Calling mpoption with no arguments returns the default options struct, the struct used if none is explicitly supplied. Calling it with a set of name and value pairs modifies the default vector.

For example, the following code runs a power flow on the 300 -bus example in case300.m using the fast-decoupled (XB version) algorithm, with verbose printing of the algorithm progress, but suppressing all of the pretty-printed output.

```
>> mpopt = mpoption('pf.alg', 'FDXB', 'verbose', 2, 'out.all', 0);
>> results = runpf('case300', mpopt);
```

To modify an existing options struct, for example, to turn the verbose option off and re-run with the remaining options unchanged, simply pass the existing options as the first argument to mpoption.

```
>> mpopt = mpoption(mpopt, 'verbose', 0);
>> results = runpf('case300', mpopt);
```

See Appendix C or type:

```
>> help mpoption
```

for more information on MATPOWER's options.

### 2.5 Documentation

There are four primary sources of documentation for Matpower. The first is this manual, which gives an overview of Matpower's capabilities and structure and describes the modeling and formulations behind the code. It can be found in your MATPOWER distribution at <MATPOWER>/docs/MATPOWER-manual.pdf and the latest version is always available at: http://www.pserc.cornell.edu/matpower/ MATPOWER-manual.pdf.

Secondly, the MOST User's Manual describes MOST and its problem formulation, features, data formats and options. It is located at $<$ MATPOWER $>/$ docs/MOST-manual.pdf in your Matpower distribution and the latest version is also available online at: http://www.pserc.cornell.edu/matpower/MOST-manual.pdf.

The Matpower Online Function Reference is the third source of documentation, allowing you to view not only the help text, but also the code itself for each MATPOWER function directly from a web browser. It is available online at: http://www.pserc.cornell.edu/matpower/docs/ref

Last, but certainly not least, is the built-in help command. As with Matlab's built-in functions and toolbox routines, you can type help followed by the name of a command or M-file to get help on that particular function. Nearly all of MatPOWER's M-files have such documentation and this should be considered the main reference for the calling options for each individual function. See Appendix D for a list of Matpower functions.

As an example, the help for runopf looks like:

```
>> help runopf
RUNOPF Runs an optimal power flow.
    [RESULTS, SUCCESS] = RUNOPF(CASEDATA, MPOPT, FNAME, SOLVEDCASE)
    Runs an optimal power flow (AC OPF by default), optionally returning
    a RESULTS struct and SUCCESS flag.
    Inputs (all are optional):
        CASEDATA : either a MATPOWER case struct or a string containing
            the name of the file with the case data (default is 'case9')
            (see also CASEFORMAT and LOADCASE)
        MPOPT : MATPOWER options struct to override default options
        can be used to specify the solution algorithm, output options
                termination tolerances, and more (see also MPOPTION).
        FNAME : name of a file to which the pretty-printed output will
            be appended
        SOLVEDCASE : name of file to which the solved case will be saved
            in MATPOWER case format (M-file will be assumed unless the
            specified name ends with '.mat')
    Outputs (all are optional):
        RESULTS : results struct, with the following fields:
            (all fields from the input MATPOWER case, i.e. bus, branch,
                gen, etc., but with solved voltages, power flows, etc.)
            order - info used in external <-> internal data conversion
            et - elapsed time in seconds
            success - success flag, 1 = succeeded, 0 = failed
            (additional OPF fields, see OPF for details)
        SUCCESS : the success flag can additionally be returned as
            a second output argument
    Calling syntax options:
    results = runopf;
    results = runopf(casedata);
    results = runopf(casedata, mpopt);
    results = runopf(casedata, mpopt, fname);
    results = runopf(casedata, mpopt, fname, solvedcase);
    [results, success] = runopf(...);
    Alternatively, for compatibility with previous versions of MATPOWER,
    some of the results can be returned as individual output arguments:
    [baseMVA, bus, gen, gencost, branch, f, success, et] = runopf(...);
    Example:
    results = runopf('case30');
See also RUNDCOPF, RUNUOPF.

\section*{3 Modeling}

MATPOWER employs all of the standard steady-state models typically used for power flow analysis. The AC models are described first, then the simplified DC models. Internally, the magnitudes of all values are expressed in per unit and angles of complex quantities are expressed in radians. Internally, all off-line generators and branches are removed before forming the models used to solve the power flow or optimal power flow problem. All buses are numbered consecutively, beginning at 1, and generators are reordered by bus number. Conversions to and from this internal indexing is done by the functions ext2int and int2ext. The notation in this section, as well as Sections 4 and 6 , is based on this internal numbering, with all generators and branches assumed to be in-service. Due to the strengths of the Matlab programming language in handling matrices and vectors, the models and equations are presented here in matrix and vector form.

\subsection*{3.1 Data Formats}

The data files used by Matpower are Matlab M-files or MAT-files which define and return a single Matlab struct. The M-file format is plain text that can be edited using any standard text editor. The fields of the struct are baseMVA, bus, branch, gen and optionally gencost, where baseMVA is a scalar and the rest are matrices. In the matrices, each row corresponds to a single bus, branch, or generator. The columns are similar to the columns in the standard IEEE CDF and PTI formats. The number of rows in bus, branch and gen are \(n_{b}, n_{l}\) and \(n_{g}\), respectively. If present, gencost has either \(n_{g}\) or \(2 n_{g}\) rows, depending on whether it includes costs for reactive power or just real power. Full details of the MATPOWER case format are documented in Appendix B and can be accessed from the Matlab command line by typing help caseformat.

\subsection*{3.2 Branches}

All transmission lines \({ }^{14}\), transformers and phase shifters are modeled with a common branch model, consisting of a standard \(\pi\) transmission line model, with series impedance \(z_{s}=r_{s}+j x_{s}\) and total charging susceptance \(b_{c}\), in series with an ideal phase shifting transformer. The transformer, whose tap ratio has magnitude \(\tau\) and

\footnotetext{
\({ }^{14}\) This does not include DC transmission lines. For more information the handling of DC transmission lines in Matpower, see Section 7.6.3.
}
phase shift angle \(\theta_{\text {shift }}\), is located at the from end of the branch, as shown in Figure 3-1. The parameters \(r_{s}, x_{s}, b_{c}, \tau\) and \(\theta_{\text {shift }}\) are specified directly in columns BR_R (3), BR_X (4), BR_B (5), TAP (9) and SHIFT (10), respectively, of the corresponding row of the branch matrix. \({ }^{15}\)

The complex current injections \(i_{f}\) and \(i_{t}\) at the from and to ends of the branch, respectively, can be expressed in terms of the \(2 \times 2\) branch admittance matrix \(Y_{b r}\) and the respective terminal voltages \(v_{f}\) and \(v_{t}\)
\[
\left[\begin{array}{c}
i_{f}  \tag{3.1}\\
i_{t}
\end{array}\right]=Y_{b r}\left[\begin{array}{c}
v_{f} \\
v_{t}
\end{array}\right]
\]

With the series admittance element in the \(\pi\) model denoted by \(y_{s}=1 / z_{s}\), the branch admittance matrix can be written
\[
Y_{b r}=\left[\begin{array}{cc}
\left(y_{s}+j \frac{b_{c}}{2}\right) \frac{1}{\tau^{2}} & -y_{s} \frac{1}{\tau e^{-j \theta_{\text {shift }}}}  \tag{3.2}\\
-y_{s} \frac{1}{\tau e^{j \theta_{\text {shift }}}} & y_{s}+j \frac{b_{c}}{2}
\end{array}\right] .
\]


Figure 3-1: Branch Model
If the four elements of this matrix for branch \(i\) are labeled as follows:
\[
Y_{b r}^{i}=\left[\begin{array}{cc}
y_{f f}^{i} & y_{f t}^{i}  \tag{3.3}\\
y_{t f}^{i} & y_{t t}^{i}
\end{array}\right]
\]
then four \(n_{l} \times 1\) vectors \(Y_{f f}, Y_{f t}, Y_{t f}\) and \(Y_{t t}\) can be constructed, where the \(i\)-th element of each comes from the corresponding element of \(Y_{b r}^{i}\). Furthermore, the \(n_{l} \times n_{b}\) sparse

\footnotetext{
\({ }^{15} \mathrm{~A}\) value of zero in the TAP column indicates that the branch is a transmission line and not a transformer, i.e. mathematically equivalent to a transformer with tap ratio set to 1 .
}
connection matrices \(C_{f}\) and \(C_{t}\) used in building the system admittance matrices can be defined as follows. The \((i, j)^{\text {th }}\) element of \(C_{f}\) and the \((i, k)^{\text {th }}\) element of \(C_{t}\) are equal to 1 for each branch \(i\), where branch \(i\) connects from bus \(j\) to bus \(k\). All other elements of \(C_{f}\) and \(C_{t}\) are zero.

\subsection*{3.3 Generators}

A generator is modeled as a complex power injection at a specific bus. For generator \(i\), the injection is
\[
\begin{equation*}
s_{g}^{i}=p_{g}^{i}+j q_{g}^{i} . \tag{3.4}
\end{equation*}
\]

Let \(S_{g}=P_{g}+j Q_{g}\) be the \(n_{g} \times 1\) vector of these generator injections. The MW and MVAr equivalents (before conversion to p.u.) of \(p_{g}^{i}\) and \(q_{g}^{i}\) are specified in columns PG (2) and QG (3), respectively of row \(i\) of the gen matrix. A sparse \(n_{b} \times n_{g}\) generator connection matrix \(C_{g}\) can be defined such that its \((i, j)^{\text {th }}\) element is 1 if generator \(j\) is located at bus \(i\) and 0 otherwise. The \(n_{b} \times 1\) vector of all bus injections from generators can then be expressed as
\[
\begin{equation*}
S_{g, \text { bus }}=C_{g} \cdot S_{g} . \tag{3.5}
\end{equation*}
\]

A generator with a negative injection can also be used to model a dispatchable load.

\subsection*{3.4 Loads}

Constant power loads are modeled as a specified quantity of real and reactive power consumed at a bus. For bus \(i\), the load is
\[
\begin{equation*}
s_{d}^{i}=p_{d}^{i}+j q_{d}^{i} \tag{3.6}
\end{equation*}
\]
and \(S_{d}=P_{d}+j Q_{d}\) denotes the \(n_{b} \times 1\) vector of complex loads at all buses. The MW and MVAr equivalents (before conversion to p.u.) of \(p_{d}^{i}\) and \(q_{d}^{i}\) are specified in columns PD (3) and QD (4), respectively of row \(i\) of the bus matrix. These fields can also take on negative quantities to represent fixed (e.g. distributed) generation.

Constant impedance and constant current loads are not implemented directly, but the constant impedance portions can be modeled as a shunt element described below. Dispatchable loads are modeled as negative generators and appear as negative values in \(S_{g}\).

\subsection*{3.5 Shunt Elements}

A shunt connected element such as a capacitor or inductor is modeled as a fixed impedance to ground at a bus. The admittance of the shunt element at bus \(i\) is given as
\[
\begin{equation*}
y_{s h}^{i}=g_{s h}^{i}+j b_{s h}^{i} \tag{3.7}
\end{equation*}
\]
and \(Y_{s h}=G_{s h}+j B_{s h}\) denotes the \(n_{b} \times 1\) vector of shunt admittances at all buses. The parameters \(g_{s h}^{i}\) and \(b_{s h}^{i}\) are specified in columns GS (5) and BS (6), respectively, of row \(i\) of the bus matrix as equivalent MW (consumed) and MVAr (injected) at a nominal voltage magnitude of 1.0 p.u and angle of zero.

A shunt element can also be used to model a constant impedance load and, though correctly, MATPOWER does not currently report these quantities as "load".

\subsection*{3.6 Network Equations}

For a network with \(n_{b}\) buses, all constant impedance elements of the model are incorporated into a complex \(n_{b} \times n_{b}\) bus admittance matrix \(Y_{\text {bus }}\) that relates the complex nodal current injections \(I_{\text {bus }}\) to the complex node voltages \(V\) :
\[
\begin{equation*}
I_{\mathrm{bus}}=Y_{\mathrm{bus}} V \tag{3.8}
\end{equation*}
\]

Similarly, for a network with \(n_{l}\) branches, the \(n_{l} \times n_{b}\) system branch admittance matrices \(Y_{f}\) and \(Y_{t}\) relate the bus voltages to the \(n_{l} \times 1\) vectors \(I_{f}\) and \(I_{t}\) of branch currents at the from and to ends of all branches, respectively:
\[
\begin{align*}
I_{f} & =Y_{f} V  \tag{3.9}\\
I_{t} & =Y_{t} V . \tag{3.10}
\end{align*}
\]

If [.] is used to denote an operator that takes an \(n \times 1\) vector and creates the corresponding \(n \times n\) diagonal matrix with the vector elements on the diagonal, these system admittance matrices can be formed as follows:
\[
\begin{align*}
Y_{f} & =\left[Y_{f f}\right] C_{f}+\left[Y_{f t}\right] C_{t}  \tag{3.11}\\
Y_{t} & =\left[Y_{t f}\right] C_{f}+\left[Y_{t t} C_{t}\right.  \tag{3.12}\\
Y_{\text {bus }} & =C_{f}^{\top} Y_{f}+C_{t}^{\top} Y_{t}+\left[Y_{s h}\right] . \tag{3.13}
\end{align*}
\]

The current injections of (3.8)-(3.10) can be used to compute the corresponding
complex power injections as functions of the complex bus voltages \(V\) :
\[
\begin{align*}
S_{\mathrm{bus}}(V) & =[V] I_{\mathrm{bus}}^{*}=[V] Y_{\mathrm{bus}}^{*} V^{*}  \tag{3.14}\\
S_{f}(V) & =\left[C_{f} V\right] I_{f}^{*}=\left[C_{f} V\right] Y_{f}^{*} V^{*}  \tag{3.15}\\
S_{t}(V) & =\left[C_{t} V\right] I_{t}^{*}=\left[C_{t} V\right] Y_{t}^{*} V^{*} . \tag{3.16}
\end{align*}
\]

The nodal bus injections are then matched to the injections from loads and generators to form the AC nodal power balance equations, expressed as a function of the complex bus voltages and generator injections in complex matrix form as
\[
\begin{equation*}
g_{S}\left(V, S_{g}\right)=S_{\mathrm{bus}}(V)+S_{d}-C_{g} S_{g}=0 \tag{3.17}
\end{equation*}
\]

\subsection*{3.7 DC Modeling}

The DC formulation [17] is based on the same parameters, but with the following three additional simplifying assumptions.
- Branches can be considered lossless. In particular, branch resistances \(r_{s}\) and charging capacitances \(b_{c}\) are negligible:
\[
\begin{equation*}
y_{s}=\frac{1}{r_{s}+j x_{s}} \approx \frac{1}{j x_{s}}, \quad b_{c} \approx 0 \tag{3.18}
\end{equation*}
\]
- All bus voltage magnitudes are close to 1 p.u.
\[
\begin{equation*}
v_{i} \approx e^{j \theta_{i}} \tag{3.19}
\end{equation*}
\]
- Voltage angle differences across branches are small enough that
\[
\begin{equation*}
\sin \left(\theta_{f}-\theta_{t}-\theta_{\text {shift }}\right) \approx \theta_{f}-\theta_{t}-\theta_{\text {shift }} \tag{3.20}
\end{equation*}
\]

Substituting the first set of assumptions regarding branch parameters from (3.18), the branch admittance matrix in (3.2) approximates to
\[
Y_{b r} \approx \frac{1}{j x_{s}}\left[\begin{array}{cc}
\frac{1}{\tau^{2}} & -\frac{1}{\tau e^{-j \theta_{\text {shift }}}}  \tag{3.21}\\
-\frac{1}{\tau e^{j s_{\text {shift }}}} & 1
\end{array}\right] .
\]

Combining this and the second assumption with (3.1) yields the following approximation for \(i_{f}\) :
\[
\begin{align*}
i_{f} & \approx \frac{1}{j x_{s}}\left(\frac{1}{\tau^{2}} e^{j \theta_{f}}-\frac{1}{\tau e^{-j \theta_{\text {shift }}}} e^{j \theta_{t}}\right) \\
& =\frac{1}{j x_{s} \tau}\left(\frac{1}{\tau} e^{j \theta_{f}}-e^{j\left(\theta_{t}+\theta_{\text {shift }}\right)}\right) \tag{3.22}
\end{align*}
\]

The approximate real power flow is then derived as follows, first applying (3.19) and (3.22), then extracting the real part and applying (3.20).
\[
\begin{align*}
p_{f} & =\Re\left\{s_{f}\right\} \\
& =\Re\left\{v_{f} \cdot i_{f}^{*}\right\} \\
& \approx \Re\left\{e^{j \theta_{f}} \cdot \frac{j}{x_{s} \tau}\left(\frac{1}{\tau} e^{-j \theta_{f}}-e^{-j\left(\theta_{t}+\theta_{\text {shift }}\right)}\right)\right\} \\
& =\Re\left\{\frac{j}{x_{s} \tau}\left(\frac{1}{\tau}-e^{j\left(\theta_{f}-\theta_{t}-\theta_{\text {shift }}\right)}\right)\right\} \\
& =\Re\left\{\frac{1}{x_{s} \tau}\left[\sin \left(\theta_{f}-\theta_{t}-\theta_{\text {shift }}\right)+j\left(\frac{1}{\tau}-\cos \left(\theta_{f}-\theta_{t}-\theta_{\text {shift }}\right)\right)\right]\right\} \\
& \approx \frac{1}{x_{s} \tau}\left(\theta_{f}-\theta_{t}-\theta_{\text {shift }}\right) \tag{3.23}
\end{align*}
\]

As expected, given the lossless assumption, a similar derivation for the power injection at the to end of the line leads to leads to \(p_{t}=-p_{f}\).

The relationship between the real power flows and voltage angles for an individual branch \(i\) can then be summarized as
\[
\left[\begin{array}{c}
p_{f}  \tag{3.24}\\
p_{t}
\end{array}\right]=B_{b r}^{i}\left[\begin{array}{c}
\theta_{f} \\
\theta_{t}
\end{array}\right]+P_{\text {shift }}^{i}
\]
where
\[
\begin{gathered}
B_{b r}^{i}=b_{i}\left[\begin{array}{rr}
1 & -1 \\
-1 & 1
\end{array}\right], \\
P_{\text {shift }}^{i}=\theta_{\text {shift }}^{i} b_{i}\left[\begin{array}{r}
-1 \\
1
\end{array}\right]
\end{gathered}
\]
and \(b_{i}\) is defined in terms of the series reactance \(x_{s}^{i}\) and tap ratio \(\tau^{i}\) for branch \(i\) as
\[
b_{i}=\frac{1}{x_{s}^{i} \tau^{i}}
\]

For a shunt element at bus \(i\), the amount of complex power consumed is
\[
\begin{align*}
s_{s h}^{i} & =v_{i}\left(y_{s h}^{i} v_{i}\right)^{*} \\
& \approx e^{j \theta_{i}}\left(g_{s h}^{i}-j b_{s h}^{i}\right) e^{-j \theta_{i}} \\
& =g_{s h}^{i}-j b_{s h}^{i} . \tag{3.25}
\end{align*}
\]

So the vector of real power consumed by shunt elements at all buses can be approximated by
\[
\begin{equation*}
P_{s h} \approx G_{s h} . \tag{3.26}
\end{equation*}
\]

With a DC model, the linear network equations relate real power to bus voltage angles, versus complex currents to complex bus voltages in the AC case. Let the \(n_{l} \times 1\) vector \(B_{f f}\) be constructed similar to \(Y_{f f}\), where the \(i\)-th element is \(b_{i}\) and let \(P_{f, \text { shift }}\) be the \(n_{l} \times 1\) vector whose \(i\)-th element is equal to \(-\theta_{\text {shift }}^{i} b_{i}\). Then the nodal real power injections can be expressed as a linear function of \(\Theta\), the \(n_{b} \times 1\) vector of bus voltage angles
\[
\begin{equation*}
P_{\mathrm{bus}}(\Theta)=B_{\mathrm{bus}} \Theta+P_{\mathrm{bus}, \text { shift }} \tag{3.27}
\end{equation*}
\]
where
\[
\begin{equation*}
P_{\mathrm{bus}, \text { shift }}=\left(C_{f}-C_{t}\right)^{\top} P_{f, \text { shift }} . \tag{3.28}
\end{equation*}
\]

Similarly, the branch flows at the from ends of each branch are linear functions of the bus voltage angles
\[
\begin{equation*}
P_{f}(\Theta)=B_{f} \Theta+P_{f, \text { shift }} \tag{3.29}
\end{equation*}
\]
and, due to the lossless assumption, the flows at the to ends are given by \(P_{t}=-P_{f}\). The construction of the system \(B\) matrices is analogous to the system \(Y\) matrices for the AC model:
\[
\begin{align*}
B_{f} & =\left[B_{f f}\right]\left(C_{f}-C_{t}\right)  \tag{3.30}\\
B_{\mathrm{bus}} & =\left(C_{f}-C_{t}\right)^{\top} B_{f} . \tag{3.31}
\end{align*}
\]

The DC nodal power balance equations for the system can be expressed in matrix form as
\[
\begin{equation*}
g_{P}\left(\Theta, P_{g}\right)=B_{\mathrm{bus}} \Theta+P_{\mathrm{bus}, \text { shift }}+P_{d}+G_{s h}-C_{g} P_{g}=0 \tag{3.32}
\end{equation*}
\]

\section*{4 Power Flow}

The standard power flow or loadflow problem involves solving for the set of voltages and flows in a network corresponding to a specified pattern of load and generation. MATPOWER includes solvers for both AC and DC power flow problems, both of which involve solving a set of equations of the form
\[
\begin{equation*}
g(x)=0, \tag{4.1}
\end{equation*}
\]
constructed by expressing a subset of the nodal power balance equations as functions of unknown voltage quantities.

All of MATPOWER's solvers exploit the sparsity of the problem and, except for Gauss-Seidel, scale well to very large systems. Currently, none of them include any automatic updating of transformer taps or other techniques to attempt to satisfy typical optimal power flow constraints, such as generator, voltage or branch flow limits.

\subsection*{4.1 AC Power Flow}

In Matpower, by convention, a single generator bus is typically chosen as a reference bus to serve the roles of both a voltage angle reference and a real power slack. The voltage angle at the reference bus has a known value, but the real power generation at the slack bus is taken as unknown to avoid overspecifying the problem. The remaining generator buses are typically classified as PV buses, with the values of voltage magnitude and generator real power injection given. These are specified in the VG (6) and PG (3) columns of the gen matrix, respectively. Since the loads \(P_{d}\) and \(Q_{d}\) are also given, all non-generator buses are classified as PQ buses, with real and reactive injections fully specified, taken from the PD (3) and QD (4) columns of the bus matrix. Let \(\mathcal{I}_{\text {ref }}, \mathcal{I}_{\mathrm{PV}}\) and \(\mathcal{I}_{\mathrm{PQ}}\) denote the sets of bus indices of the reference bus, PV buses and PQ buses, respectively. The bus type classification is specified in the Matpower case file in the BUS_TYPE column (2) of the bus matrix. Any isolated buses must be identified as such in this column as well.

In the traditional formulation of the AC power flow problem, the power balance equation in (3.17) is split into its real and reactive components, expressed as functions of the voltage angles \(\Theta\) and magnitudes \(V_{m}\) and generator injections \(P_{g}\) and \(Q_{g}\), where the load injections are assumed constant and given:
\[
\begin{align*}
g_{P}\left(\Theta, V_{m}, P_{g}\right) & =P_{\mathrm{bus}}\left(\Theta, V_{m}\right)+P_{d}-C_{g} P_{g} \tag{4.2}
\end{align*}=0.0 .
\]

For the AC power flow problem, the function \(g(x)\) from (4.1) is formed by taking the left-hand side of the real power balance equations (4.2) for all non-slack buses and the reactive power balance equations (4.3) for all PQ buses and plugging in the reference angle, the loads and the known generator injections and voltage magnitudes:
\[
g(x)=\left[\begin{array}{cl}
g_{P}^{\{i\}}\left(\Theta, V_{m}, P_{g}\right)  \tag{4.4}\\
g_{Q}^{\{j\}}\left(\Theta, V_{m}, Q_{g}\right)
\end{array}\right] \quad \begin{array}{ll}
\forall i \in \mathcal{I}_{\mathrm{PV}} \cup \mathcal{I}_{\mathrm{PQ}} \\
\forall j \in \mathcal{I}_{\mathrm{PQ}}
\end{array}
\]

The vector \(x\) consists of the remaining unknown voltage quantities, namely the voltage angles at all non-reference buses and the voltage magnitudes at PQ buses:
\[
x=\left[\begin{array}{c}
\theta_{\{i\}}  \tag{4.5}\\
v_{m}^{j j\}}
\end{array}\right] \quad \begin{aligned}
& \forall i \notin \mathcal{I}_{\text {ref }} \\
& \forall j \in \mathcal{I}_{\mathrm{PQ}} .
\end{aligned}
\]

This yields a system of nonlinear equations with \(n_{p v}+2 n_{p q}\) equations and unknowns, where \(n_{p v}\) and \(n_{p q}\) are the number of PV and PQ buses, respectively. After solving for \(x\), the remaining real power balance equation can be used to compute the generator real power injection at the slack bus. Similarly, the remaining \(n_{p v}+1\) reactive power balance equations yield the generator reactive power injections.

Matpower includes four different algorithms for solving the general AC power flow problem. \({ }^{16}\) The default solver is based on a standard Newton's method [8] using a polar form and a full Jacobian updated at each iteration. Each Newton step involves computing the mismatch \(g(x)\), forming the Jacobian based on the sensitivities of these mismatches to changes in \(x\) and solving for an updated value of \(x\) by factorizing this Jacobian. This method is described in detail in many textbooks.

Also included are solvers based on variations of the fast-decoupled method [9], specifically, the XB and BX methods described in [10]. These solvers greatly reduce the amount of computation per iteration, by updating the voltage magnitudes and angles separately based on constant approximate Jacobians which are factored only once at the beginning of the solution process. These per-iteration savings, however, come at the cost of more iterations.

The fourth algorithm is the standard Gauss-Seidel method from Glimm and Stagg [11]. It has numerous disadvantages relative to the Newton method and is included primarily for academic interest.

By default, the AC power flow solvers simply solve the problem described above, ignoring any generator limits, branch flow limits, voltage magnitude limits, etc. However, there is an option (pf.enforce_q-lims) that allows for the generator reactive

\footnotetext{
\({ }^{16}\) Three more that are specific to radial networks typical of distribution systems are described in Section 4.3.
}
power limits to be respected at the expense of the voltage setpoint. This is done in a rather brute force fashion by adding an outer loop around the AC power flow solution. If any generator has a violated reactive power limit, its reactive injection is fixed at the limit, the corresponding bus is converted to a PQ bus and the power flow is solved again. This procedure is repeated until there are no more violations. Note that this option is based solely on the QMAX and QMIN parameters for the generator, from columns 4 and 5 of the gen matrix, and does not take into account the trapezoidal generator capability curves described in Section 6.4.3 and specifed in columns PC1-QC2MAX (11-16). Note also that this option affects generators even if the bus they are attached to is already of type PQ.

\subsection*{4.2 DC Power Flow}

For the DC power flow problem [17], the vector \(x\) consists of the set of voltage angles at non-reference buses
\[
\begin{equation*}
x=\left[\theta_{\{i\}}\right], \quad \forall i \notin \mathcal{I}_{\mathrm{ref}} \tag{4.6}
\end{equation*}
\]
and (4.1) takes the form
\[
\begin{equation*}
B_{d c} x-P_{d c}=0 \tag{4.7}
\end{equation*}
\]
where \(B_{d c}\) is the \(\left(n_{b}-1\right) \times\left(n_{b}-1\right)\) matrix obtained by simply eliminating from \(B_{\text {bus }}\) the row and column corresponding to the slack bus and reference angle, respectively. Given that the generator injections \(P_{g}\) are specified at all but the slack bus, \(P_{d c}\) can be formed directly from the non-slack rows of the last four terms of (3.32).

The voltage angles in \(x\) are computed by a direct solution of the set of linear equations. The branch flows and slack bus generator injection are then calculated directly from the bus voltage angles via (3.29) and the appropriate row in (3.32), respectively.

\subsection*{4.3 Distribution Power Flow}

Distribution systems are different from transmission systems in a number of respects, such as the \(x_{s} / r_{s}\) branch ratio, magnitudes of \(x_{s}\) and \(r_{s}\) and most importantly the typically radial structure. Due to these differences, a number of power flow solution methods have been developed to account for the specific nature of distribution systems and most widely used are the backward/forward sweep methods \([12,13]\). Matpower includes an additional three AC power flow methods that are specific to radial networks.

\subsection*{4.3.1 Radial Power Flow}

When solving radial distribution networks it is practical to number branches with numbers that are equal to the receiving bus numbers. An example is given in Figure 4-1, where branches are drawn black. Furthermore, the oriented branch ordering [14] offers a possibility for fast and efficient backward/forward sweeps. All branches are always oriented from the sending bus to the receiving bus and the only requirement is that the sending bus number should be smaller than the receiving bus number. This means that \(i<k\) for branch \(i-k\). The indices of the sending nodes of branches are stored in vector \(F\) such that \(i=f_{k}\).


Figure 4-1: Oriented Ordering
As usual, the supply bus (slack bus) is given index 1, meaning that branch indices should go from 2 to \(n_{b}\) which is the number of buses in the network. Introducing a fictitious branch with index 1 and zero impedance, given with dashed black line in Figure 4-1, the number of branches \(n_{l}\) becomes equal to the number of buses \(n_{b}\).

For the example of Figure \(4-1\) vector \(F\) is the following
\[
F=\left[\begin{array}{lllllll}
0 & 1 & 2 & 3 & 3 & 2 & 4
\end{array}\right]^{\top}
\]
and it offers an easy way to follow the path between any bus and bus 0 . If we consider bus 4 , the path to the slack bus consists of following branches: branch 4 , since considered bus is their receiving bus; branch 3 , since \(f_{4}=3\); branch 2 , since \(f_{3}=2\); and branch 1 , since \(f_{2}=1\).

The representation of branch \(k\), connecting buses \(i\) and \(k\), is given in Figure 4-2, where it is modeled with its serial impedance \(z_{s}^{k}\). At both ends there are load demands \(s_{d}^{i}\) and \(s_{d}^{k}\), and shunt admittances \(y_{d}^{i}\) and \(y_{d}^{k}\) comprised of admittances due capacitance of all lines and shunt elements connected to buses \(i\) and \(k\)
\[
y_{d}^{k}=j\left(\sum b_{\text {lines }}^{k}+\sum b_{\text {shunt }}^{k}\right)
\]


Figure 4-2: Branch Representation: branch \(k\) between buses \(i\) (sending) and \(k\) (receiving) and load demand and shunt admittances at both buses

\subsection*{4.3.2 Current Summation Method}

The voltage calculation procedure with the Current Summation Method is performed in 5 steps as follows \([12,13]\).
1. Set all voltages to 1 p.u. (flat start). Set iteration count \(\nu=1\).
2. Set branch current flow equal to the sum of current of the demand at receiving end \(\left(s_{d}^{k}\right)\) and the current drawn in the admittance \(\left(y_{d}^{k}\right)\) connected to bus \(k\)
\[
\begin{equation*}
j_{b}^{k}=\left(\frac{s_{d}^{k}}{v_{k}}\right)^{*}+y_{d}^{k} \cdot v_{k}, \quad k=1,2, \ldots, n_{b} \tag{4.8}
\end{equation*}
\]
3. Backward sweep: Perform current summation, starting from the branch with the biggest index and heading towards the branch whose index is equal to 1 . The current of branch \(k\) is added to the current of the branch whose index is equal to \(i=f_{k}\).
\[
\begin{equation*}
j_{b, \text { new }}^{i}=j_{b}^{i}+j_{b}^{k}, \quad k=n_{l}, n_{l}-1, \ldots, 2 \tag{4.9}
\end{equation*}
\]
4. Forward sweep: The receiving end bus voltages are calculated with known branch currents and sending bus voltages.
\[
\begin{equation*}
v_{k}=v_{i}-z_{s}^{k} \cdot j_{b}^{k}, \quad k=2,3, \ldots, n_{l} \tag{4.10}
\end{equation*}
\]
5. Compare voltages in iteration \(\nu\) with the corresponding ones from iteration \(\nu-1\). If the maximum difference in magnitude is less than the specified tolerance
\[
\begin{equation*}
\max _{i=1 \ldots n_{b}}\left\{\left|v_{i}^{\nu}-v_{i}^{\nu-1}\right|\right\}<\varepsilon \tag{4.11}
\end{equation*}
\]
the procedure is finished. Otherwise go to step 2.

\subsection*{4.3.3 Power Summation Method}

The voltage calculation procedure with the Power Summation Method is performed in 5 steps as follows [14].
1. Set all voltages to 1 p.u. (flat start). Set iteration count \(\nu=1\).
2. Set receiving end branch flow equal to the sum of the demand at receiving end \(\left(s_{d}^{k}\right)\) and the power drawn in the admittance \(\left(y_{d}^{k}\right)\) connected to bus \(k\)
\[
\begin{equation*}
s_{t}^{k}=s_{d}^{k}+\frac{\left(y_{d}^{k}\right)^{*}}{v_{k}^{2}}, \quad k=1,2, \ldots, n_{b} . \tag{4.12}
\end{equation*}
\]
3. Backward sweep: Calculate sending end branch power flows as a sum of receiving end branch power flows and branch losses via (4.13). Perform power summation, starting from the branch with the biggest index and heading towards the branch whose index is equal to 1 . The sending power of branch \(k\) is added to the receiving power of the branch whose index is equal to \(i=f_{k}\) as in (4.14).
\[
\begin{array}{rlrl}
s_{f}^{k} & =s_{t}^{k}+z_{s}^{k} \cdot\left|\frac{s_{t}^{k}}{v_{k}}\right|^{2} & k & =n_{l}, n_{l}-1, \ldots, 2 \\
s_{t, \text { new }}^{i} & =s_{t}^{i}+s_{f}^{k} & k=n_{l}, n_{l}-1, \ldots, 2 \tag{4.14}
\end{array}
\]
4. Forward sweep: The receiving end bus voltages are calculated with known sending powers and voltages.
\[
\begin{equation*}
v_{k}=v_{i}-z_{s}^{k} \cdot\left(\frac{s_{f}^{k}}{v_{i}}\right)^{*} \quad k=2,3, \ldots, n_{l} \tag{4.15}
\end{equation*}
\]
5. Compare voltages in iteration \(\nu\) with the corresponding ones from iteration \(\nu-1\), using (4.11). If the maximum difference in magnitude is less than the specified tolerance the procedure is finished. Otherwise go to step 2.

\subsection*{4.3.4 Admittance Summation Method}

For each node, besides the known admittance \(y_{d}^{k}\), we define the admittance \(y_{e}^{k}\) as the driving point admittance of the part of the network fed by node \(k\), including shunt
admittance \(y_{d}^{k}\). We also define an equivalent current generator \(j_{e}^{k}\) for the part of the network fed by node \(k\). The current of this generator consists of all load currents fed by node \(k\). The process of calculation of bus voltages with the admittance summation method consists of the following 5 steps [15].
1. Set all voltages to 1 p.u. (flat start). Set iteration count \(\nu=1\). Set initial values
\[
\begin{equation*}
y_{e}^{k}=y_{d}^{k}, \quad k=1,2, \ldots, n_{b} . \tag{4.16}
\end{equation*}
\]
2. For each node \(k\), calculate equivalent admittance \(y_{e}^{k}\). Perform admittance summation, starting from the branch with the biggest index and heading towards the branch whose index is equal to 1 . The driving point admittance of branch \(k\) is added to the driving point admittance of the branch whose index is equal to \(i=f_{k}\) as in (4.18).
\[
\begin{array}{rlrl}
d_{b}^{k} & =\frac{1}{1+z_{s}^{k} \cdot y_{e}^{k}} & k & =n_{l}, n_{l}-1, \ldots, 2 \\
y_{e, \text { new }}^{i} & =y_{e}^{i}+d_{b}^{k} \cdot y_{e}^{k} & k & =n_{l}, n_{l}-1, \ldots, 2 \tag{4.18}
\end{array}
\]
3. Backward sweep: For each node \(k\) calculate equivalent current generator \(j_{e}^{k}\), first set it equal to load current \(j_{d}^{k}\) and perform current summation over equivalent admittances using factor \(d_{b}^{k}\) as in (4.17).
\[
\begin{array}{rlrl}
j_{e}^{k} & =j_{d}^{k}=\left(\frac{s_{d}^{k}}{v_{k}}\right)^{*} & k & =n_{l}, n_{l}-1, \ldots, 2 \\
j_{e, \text { new }}^{i} & =j_{e}^{i}+d_{b}^{k} \cdot j_{e}^{k} & k & =n_{l}, n_{l}-1, \ldots, 2 \tag{4.20}
\end{array}
\]
4. Forward sweep: The receiving end bus voltages are calculated with known equivalent current generators and sending bus voltages.
\[
\begin{equation*}
v_{k}=d_{b}^{k} \cdot\left(v_{i}-z_{s}^{k} \cdot j_{e}^{k}\right) \quad k=2,3, \ldots, n_{l} \tag{4.21}
\end{equation*}
\]
5. Compare voltages in iteration \(\nu\) with the corresponding ones from iteration \(\nu-1\), using (4.11). If the maximum difference in magnitude is less than the specified tolerance the procedure is finished. Otherwise go to step 3.

\subsection*{4.3.5 Handling PV Buses}

The methods explained in the previous three subsections are applicable to radial networks without loops and PV buses. These methods can be used to solve the power flow problem in weakly meshed networks if a compensation procedure based on Thevenin equivalent impedance matrix is added [12,13]. In [14] a voltage correction procedure is added to the process.

The list of branches is expanded by a set of \(n_{p v}\) fictitious links, corresponding to the PV nodes. Each of these links starts at the slack bus and ends at a corresponding PV bus, thus forming a loop in the network. A fictitious link going to bus \(k\) is represented by a voltage generator with a voltage magnitude equal to the specified voltage magnitude at bus \(k\). Its phase angle is equal to the calculated phase angle at bus \(k\).

A loop impedance matrix \(Z_{l}\) is formed for the loops made by the fictitious links and it has the following properties
- Element \(z_{l}^{m m}\) is equal to the sum of branch impedances off all branches related to loop \(m\),
- Element \(z_{l}^{m k}\) is equal to the sum of branch impedances of mutual branches of loops \(m\) and \(k\).

As an illustration, in Figure 4-1 there a two PV generators at buses 5 and 7. The fictitious links and loops orientation are drawn in red. The Thevenin matrix for this case is
\[
Z_{l}=\left[\begin{array}{cc}
z_{b}^{2}+z_{b}^{3}+z_{b}^{5} & z_{b}^{2}+z_{b}^{3} \\
z_{b}^{2}+z_{b}^{3} & z_{b}^{2}+z_{b}^{3}+z_{b}^{4}+z_{b}^{7}
\end{array}\right] .
\]

First column elements are equal to PV bus voltages when the current injection at bus 5 is 1 p.u. and \(v_{1}=0\). Bus voltages can be calculated with the current summation method in a single iteration. By repeating the procedure for bus 7 one can calculate the elements of second column.

By breaking all links the network becomes radial [13] and the three backward/forward sweep methods are applicable. Since all link are fictitious, only the injected reactive power at their receiving bus \(m\) is determined [14] by the following equation
\[
\begin{equation*}
\Delta q_{p v}^{m}=\Delta d_{p v}^{m} \frac{v_{m}^{2}}{\Re\left\{v_{m}\right\}} \tag{4.22}
\end{equation*}
\]
which is practically an increment in reactive power injection of the corresponding PV generator for the current iteration.

The incremental changes of the imaginary part of PV generator current \(\Delta d_{p v}^{m}\) can be obtained by solving the matrix equation
\[
\begin{equation*}
\Im\left\{Z_{l}\right\} \cdot \Delta D_{p v}=\Delta E_{p v} \tag{4.23}
\end{equation*}
\]
where
\[
\begin{equation*}
\Delta e_{p v}^{m}=\left(\frac{v_{g}^{m}}{\left|v_{m}\right|}-1\right) \cdot \Re\left\{v_{m}\right\}, \quad m=1,2, \ldots, n_{p v} \tag{4.24}
\end{equation*}
\]

In order to ensure \(90^{\circ}\) phase difference between voltage and current at PV generators in [16] it was suggested to calculate the real part of PV generator current as
\[
\begin{equation*}
\Delta c_{p v}^{m}=\Delta d_{p v}^{m} \frac{\Im\left\{v_{m}\right\}}{\Re\left\{v_{m}\right\}} . \tag{4.25}
\end{equation*}
\]

In such a way the PV generator will inject purely reactive power, as it is supposed to do. Its active power is added before as a negative load.

Before proceeding with the next iteration, the bus voltage corrections are calculated. In order to do that, the radial network is solved by applying incremental current changes \(\Delta I_{p v}=\Delta C_{p v}+j \Delta D_{p v}\) at the PV buses as excitations and setting \(v_{1}=0\). After the backward/forward sweep is performed with the current summation method, the voltage corrections at all buses are known. They are added to the latest voltages in order to obtain the new bus voltages, which are used in the next iteration [14].

\section*{4.4 runpf}

In Matpower, a power flow is executed by calling runpf with a case struct or case file name as the first argument (casedata). In addition to printing output to the screen, which it does by default, runpf optionally returns the solution in a results struct.
```

>> results = runpf(casedata);

```

The results struct is a superset of the input MATPOWER case struct mpc, with some additional fields as well as additional columns in some of the existing data fields. The solution values are stored as shown in Table 4-1.

Additional optional input arguments can be used to set options (mpopt) and provide file names for saving the pretty printed output (fname) or the solved case data (solvedcase).

Table 4-1: Power Flow Results
\begin{tabular}{ll}
\hline name & description \\
\hline results.success & success flag, \(1=\) succeeded, \(0=\) failed \\
results.et & computation time required for solution \\
results.iterations & number of iterations required for solution \\
results.order & see ext2int help for details on this field \\
results.bus (:, VM \()^{\dagger}\) & bus voltage magnitudes \\
results.bus (:, VA) & bus voltage angles \\
results.gen(:, PG) & generator real power injections \\
results.gen(:, QG \()^{\dagger}\) & generator reactive power injections \\
results.branch(:, PF) & real power injected into "from" end of branch \\
results.branch(:, PT) & real power injected into "to" end of branch \\
results.branch(:, QF \()^{\dagger}\) & reactive power injected into "from" end of branch \\
results.branch(:, QT \()^{\dagger}\) & reactive power injected into "to" end of branch \\
\hline
\end{tabular}
\({ }^{\dagger} \mathrm{AC}\) power flow only.
```

>> results = runpf(casedata, mpopt, fname, solvedcase);

```

The options that control the power flow simulation are listed in Table 4-2 and those controlling the output printed to the screen in Table 4-3.

By default, runpf solves an AC power flow problem using a standard Newton's method solver. To run a DC power flow, the model option must be set to 'DC'. For convenience, Matpower provides a function rundcpf which is simply a wrapper that sets the model option to 'DC' before calling runpf.

Internally, the runpf function does a number of conversions to the problem data before calling the appropriate solver routine for the selected power flow algorithm. This external-to-internal format conversion is performed by the ext2int function, described in more detail in Section 7.3.1, and includes the elimination of out-of-service equipment, the consecutive renumbering of buses and the reordering of generators by increasing bus number. All computations are done using this internal indexing. When the simulation has completed, the data is converted back to external format by int2ext before the results are printed and returned.

\subsection*{4.5 Linear Shift Factors}

The DC power flow model can also be used to compute the sensitivities of branch flows to changes in nodal real power injections, sometimes called injection shift factors (ISF) or generation shift factors [17]. These \(n_{l} \times n_{b}\) sensitivity matrices, also called

Table 4-2: Power Flow Options
\begin{tabular}{|c|c|c|}
\hline name & default & description \\
\hline model & ' AC' & AC vs. DC modeling for power flow and OPF formulation 'AC' - use AC formulation and corresponding alg options 'DC' - use DC formulation and corresponding alg options \\
\hline pf.alg & ' NR ' & \begin{tabular}{l}
AC power flow algorithm: \\
'NR' - Newtons's method \\
'FDXB' - Fast-Decoupled (XB version) \\
' FDBX' - Fast-Decouple (BX version) \\
'GS' - Gauss-Seidel \\
'PQSUM' - Power Summation (radial networks only) \\
'ISUM' - Current Summation (radial networks only) \\
'YSUM' - Admittance Summation (radial networks only)
\end{tabular} \\
\hline pf.tol & \(10^{-8}\) & termination tolerance on per unit P and Q dispatch \\
\hline pf.nr.max_it & 10 & maximum number of iterations for Newton's method \\
\hline pf.nr.lin_solver & ' ' & \begin{tabular}{l}
linear solver option for mplinsolve for computing Newton update step (see mplinsolve for complete list of all options) \\
'' - default to '\' for small systems, 'LU3' for larger ones \\
' \' - built-in backslash operator \\
'LU' - explicit default LU decomposition and back substitution \\
'LU3' - 3 output arg form of lu, Gilbert-Peierls algorithm with approximate minimum degree (AMD) reordering \\
'LU4' - 4 output arg form of \(1 u\), UMFPACK solver (same as 'LU')
\end{tabular} \\
\hline pf.fd.max_it & 30 & 'LU5' - 5 output arg form of 1 u , UMFPACK solver w/row scaling maximum number of iterations for fast-decoupled method \\
\hline pf.gs.max_it & 1000 & maximum number of iterations for Gauss-Seidel method \\
\hline pf.radial.max_it & 20 & maximum number of iterations for radial power flow methods \\
\hline pf.radial.vcorr & 0 & \begin{tabular}{l}
perform voltage correction procedure in distribution power flow 0 - do not perform voltage correction \\
1 - perform voltage correction
\end{tabular} \\
\hline pf.enforce_q_lims & 0 & \begin{tabular}{l}
enforce gen reactive power limits at expense of \(\left|V_{m}\right|\) \\
0 - do not enforce limits \\
1 - enforce limits, simultaneous bus type conversion \\
2 - enforce limits, one-at-a-time bus type conversion
\end{tabular} \\
\hline
\end{tabular}
power transfer distribution factors or PTDFs, carry an implicit assumption about the slack distribution. If \(H\) is used to denote a PTDF matrix, then the element in row \(i\) and column \(j, h_{i j}\), represents the change in the real power flow in branch \(i\) given a unit increase in the power injected at bus \(j\), with the assumption that the additional unit of power is extracted according to some specified slack distribution:
\[
\begin{equation*}
\Delta P_{f}=H \Delta P_{\mathrm{bus}} \tag{4.26}
\end{equation*}
\]

Table 4-3: Power Flow Output Options
\begin{tabular}{|c|c|c|}
\hline name & default & description \\
\hline \multirow[t]{5}{*}{verbose} & \multirow[t]{5}{*}{1} & amount of progress info to be printed \\
\hline & & 0 - print no progress info \\
\hline & & 1 - print a little progress info \\
\hline & & 2 - print a lot of progress info \\
\hline & & 3 - print all progress info \\
\hline \multirow[t]{4}{*}{out.all} & \multirow[t]{4}{*}{-1} & controls pretty-printing of results \\
\hline & & -1 - individual flags control what is printed \\
\hline & & 0 - do not print anything \({ }^{\dagger}\) \\
\hline & & \(1-\) print everything \(^{\dagger}\) \\
\hline out.sys_sum & 1 & print system summary (0 or 1 ) \\
\hline out.area_sum & 0 & print area summaries (0 or 1 ) \\
\hline out.bus & 1 & print bus detail, includes per bus gen info (0 or 1 ) \\
\hline out.branch & 1 & print branch detail (0 or 1) \\
\hline out.gen & 0 & print generator detail (0 or 1) \\
\hline out.force & 0 & print results even if success flag \(=0\) (0 or 1) \\
\hline \multirow[t]{3}{*}{out.suppress_detail} & -1 & suppress all output but system summary \\
\hline & & \(-1-\) suppress details for large systems ( \(>500\) buses) \\
\hline & & \begin{tabular}{l}
0 - do not suppress any output specified by other flags \\
1 - suppress all output except system summary section \({ }^{\dagger}\)
\end{tabular} \\
\hline
\end{tabular}
\(\dagger\) Overrides individual flags, but (in the case of out.suppress_detail) not out.all \(=1\).

This slack distribution can be expressed as an \(n_{b} \times 1\) vector \(w\) of non-negative weights whose elements sum to 1 . Each element specifies the proportion of the slack taken up at each bus. For the special case of a single slack bus \(k, w\) is equal to the vector \(e_{k}\). The corresponding PTDF matrix \(H_{k}\) can be constructed by first creating the \(n_{l} \times\left(n_{b}-1\right)\) matrix
\[
\begin{equation*}
\widetilde{H}_{k}=\widetilde{B}_{f} \cdot B_{d c}^{-1} \tag{4.27}
\end{equation*}
\]
then inserting a column of zeros at column \(k\). Here \(\widetilde{B}_{f}\) and \(B_{d c}\) are obtained from \(B_{f}\) and \(B_{\text {bus }}\), respectively, by eliminating their reference bus columns and, in the case of \(B_{d c}\), removing row \(k\) corresponding to the slack bus.

The PTDF matrix \(H_{w}\), corresponding to a general slack distribution \(w\), can be obtained from any other PTDF, such as \(H_{k}\), by subtracting \(H_{k} \cdot w\) from each column, equivalent to the following simple matrix multiplication:
\[
\begin{equation*}
H_{w}=H_{k}\left(I-w \cdot 1^{\top}\right) \tag{4.28}
\end{equation*}
\]

These same linear shift factors may also be used to compute sensitivities of branch flows to branch outages, known as line outage distribution factors or LODFs [18].

Given a PTDF matrix \(H_{w}\), the corresponding \(n_{l} \times n_{l}\) LODF matrix \(L\) can be constructed as follows, where \(l_{i j}\) is the element in row \(i\) and column \(j\), representing the change in flow in branch \(i\) (as a fraction of the initial flow in branch \(j\) ) for an outage of branch \(j\).

First, let \(H\) represent the matrix of sensitivities of branch flows to branch endpoint injections, found by multplying the PTDF matrix by the node-branch incidence matrix:
\[
\begin{equation*}
H=H_{w}\left(C_{f}-C_{t}\right)^{\top} . \tag{4.29}
\end{equation*}
\]

Here the individual elements \(h_{i j}\) represent the sensitivity of flow in branch \(i\) with respect to injections at branch \(j\) endpoints, corresponding to a simulated increase in flow in branch \(j\). Then \(l_{i j}\) can be expressed as
\[
l_{i j}=\left\{\begin{array}{cc}
\frac{h_{i j}}{1-h_{j j}} & i \neq j  \tag{4.30}\\
-1 & i=j .
\end{array}\right.
\]

Matpower includes functions for computing both the DC PTDF matrix and the corresponding LODF matrix for either a single slack bus \(k\) or a general slack distribution vector \(w\). See the help for makePTDF and makeLODF and Sections 9.5.5 and 9.5.4, respectively, for details.

\section*{5 Continuation Power Flow}

Continuation methods or branch tracing methods are used to trace a curve given an initial point on the curve. These are also called predictor-corrector methods since they involve the prediction of the next solution point and correcting the prediction to get the next point on the curve.

Consider a system of \(n\) nonlinear equations \(g(x)=0, x \in \mathbb{R}^{n}\). By adding a continuation parameter \(\lambda\) and one more equation to the system, \(x\) can be traced by varying \(\lambda\). The resulting system \(f(x, \lambda)=0\) has \(n+1\) dimensions. The additional equation is a parameterized equation which identifies the location of the current solution with respect to the previous or next solution.

The continuation process can be diagrammatically shown by (5.1).
\[
\begin{equation*}
\left(x^{j}, \lambda^{j}\right) \xrightarrow{\text { Predictor }}\left(\hat{x}^{j+1}, \hat{\lambda}^{j+1}\right) \xrightarrow{\text { Corrector }}\left(x^{j+1}, \lambda^{j+1}\right) \tag{5.1}
\end{equation*}
\]
where, \(\left(x^{j}, \lambda^{j}\right)\) represents the current solution at step \(j,\left(\hat{x}^{j+1}, \hat{\lambda}^{j+1}\right)\) is the predicted solution for the next step, and \(\left(x^{j+1}, \lambda^{j+1}\right)\) is the next solution on the curve.

Continuation methods are employed in power systems to determine steady state stability limits [19] in what is called a continuation power flow \({ }^{17}\). The limit is determined from a nose curve where the nose represents the maximum power transfer that the system can handle given a power transfer schedule. To determine the steady state loading limit, the basic power flow equations
\[
g(x)=\left[\begin{array}{c}
P(x)-P^{i n j}  \tag{5.2}\\
Q(x)-Q^{i n j}
\end{array}\right]=0,
\]
are restructured as
\[
\begin{equation*}
f(x, \lambda)=g(x)-\lambda b=0 \tag{5.3}
\end{equation*}
\]
where \(x \equiv\left(\Theta, V_{m}\right)\) and \(b\) is a vector of power transfer given by
\[
b=\left[\begin{array}{c}
P_{\text {target }}^{i n j}-P_{\text {base }}^{i n j}  \tag{5.4}\\
Q_{\text {target }}^{i n j}-Q_{\text {base }}^{i n j}
\end{array}\right] .
\]

The effects of the variation of loading or generation can be investigated using the continuation method by composing the \(b\) vector appropriately.

\footnotetext{
\({ }^{17}\) Thanks to Shrirang Abhyankar, Rui Bo, and Alexander Flueck for contributions to MatPOWER's continuation power flow feature.
}

\subsection*{5.1 Parameterization}

The values of \((x, \lambda)\) along the solution curve can parameterized in a number of ways [20,21]. Parameterization is a mathematical way of identifying each solution so that the next solution or previous solution can be quantified. Matpower includes three parameterization scheme options to quantify this relationship, detailed below, where \(\sigma\) is the continuation step size parameter.
- Natural parameterization simply uses \(\lambda\) directly as the parameter, so the new \(\lambda\) is simply the previous value plus the step size.
\[
\begin{equation*}
p^{j}(x, \lambda)=\lambda-\lambda^{j}-\sigma^{j}=0 \tag{5.5}
\end{equation*}
\]
- Arc length parameterization results in the following relationship, where the step size is equal to the 2 -norm of the distance from one solution to the next.
\[
\begin{equation*}
p^{j}(x, \lambda)=\sum_{i}\left(x_{i}-x_{i}^{j}\right)^{2}+\left(\lambda-\lambda^{j}\right)^{2}-\left(\sigma^{j}\right)^{2}=0 \tag{5.6}
\end{equation*}
\]
- Pseudo arc length parameterization [23] is MATPOWER's default parameterization scheme, where the next point \((x, \lambda)\) on the solution curve is constrained to lie in the hyperplane running through the predicted solution \(\left(\hat{x}^{j+1}, \hat{\lambda}^{j+1}\right)\) orthogonal to the tangent line from the previous corrected solution \(\left(x^{j}, \lambda^{j}\right)\). This relationship can be quantified by the function
\[
p^{j}(x, \lambda)=\left(\left[\begin{array}{c}
x  \tag{5.7}\\
\lambda
\end{array}\right]-\left[\begin{array}{c}
x^{j} \\
\lambda^{j}
\end{array}\right]\right)^{\top} \bar{z}^{j}-\sigma^{j}=0,
\]
where \(\bar{z}^{j}\) is the normalized tangent vector at \(\left(x^{j}, \lambda^{j}\right)\) and \(\sigma^{j}\) is the continuation step size parameter.

\subsection*{5.2 Predictor}

The predictor is used to produce an estimate for the next solution. The better the prediction, the faster is the convergence to the solution point. Matpower uses a tangent predictor for estimating the curve to the next solution. The tangent vector \(z^{j}=\left[\begin{array}{ll}d x & d \lambda\end{array}\right]_{j}^{\top}\) at the current solution \(\left(x^{j}, \lambda^{j}\right)\) is found by solving the linear system
\[
\left[\begin{array}{cc}
f_{x} & f_{\lambda}  \tag{5.8}\\
p_{x}^{j-1} & p_{\lambda}^{j-1}
\end{array}\right] z^{j}=\left[\begin{array}{l}
0 \\
1
\end{array}\right] .
\]

The matrix on the left-hand side is simply the standard power flow Jacobian with an additional column and row added. The extra column \(f_{\lambda}\) is simply the negative of the power transfer vector \(b\) and the extra row, required to make the system non-singular and define the magnitude of \(z^{j}\), is the derivative of the the parameterization function at the previous solution point \(p^{j-1}\).

The resulting tangent vector is then normalized
\[
\begin{equation*}
\bar{z}^{j}=\frac{z^{j}}{\left\|z^{j}\right\|_{2}} \tag{5.9}
\end{equation*}
\]
and used to compute the predicted approximation \(\left(\hat{x}^{j+1}, \hat{\lambda}^{j+1}\right)\) to the next solution \(\left(x^{j+1}, \lambda^{j+1}\right)\) using
\[
\left[\begin{array}{c}
\hat{x}^{j+1}  \tag{5.10}\\
\hat{\lambda}^{j+1}
\end{array}\right]=\left[\begin{array}{c}
x^{j} \\
\lambda^{j}
\end{array}\right]+\sigma^{j} \bar{z}^{j},
\]
where \(\sigma^{j}\) is the continuation step size.

\subsection*{5.3 Corrector}

The corrector stage finds the next solution \(\left(x^{j+1}, \lambda^{j+1}\right)\) by correcting the approximation estimated by the predictor \(\left(\hat{x}^{j+1}, \hat{\lambda}^{j+1}\right)\). Newton's method is used to find the next solution by solving the \(n+1\) dimensional system in (5.11), where one of (5.5)-(5.7) has been added as an additional constraint to the parameterized power flow equations of (5.3).
\[
\left[\begin{array}{c}
f(x, \lambda)  \tag{5.11}\\
p^{j}(x, \lambda)
\end{array}\right]=0
\]

\subsection*{5.4 Step Length Control}

Step length control is a key element affecting the computational efficiency of a continuation method. It affects the continuation method with two issues: (1) speed - how fast the corrector converges to a specified accuracy, and (2) robustness - whether the corrector converges to a true solution given a predicted point. Matpower's continuation power flow can optionally use adaptive steps, where the step size \(\sigma\) is adjusted by a scaling factor \(\alpha\) within the limits specified.
\[
\begin{equation*}
\sigma^{j+1}=\alpha^{j} \sigma^{j}, \quad \sigma_{\min } \leq \sigma^{j+1} \leq \sigma_{\max } \tag{5.12}
\end{equation*}
\]

This scaling factor \(\alpha^{j}\) for step \(j\) is limited to a maximum of 2 and is calculated from an error estimation between the predicted and corrected solutions \(\gamma^{j}\) as follows,
\[
\begin{equation*}
\alpha^{j}=1+\beta_{\mathrm{cpf}}\left(\frac{\epsilon_{\mathrm{cpf}}}{\gamma^{j}}-1\right), \quad \alpha^{j} \leq 2 \tag{5.13}
\end{equation*}
\]
where \(\beta_{\mathrm{cpf}}\) is a damping factor, \(\epsilon_{\mathrm{cpf}}\) is a specified tolerance, and \(\gamma^{j}\) is given by
\[
\begin{equation*}
\gamma^{j}=\left\|\left(x^{j+1}, \lambda^{j+1}\right)-\left(\hat{x}^{j+1}, \hat{\lambda}^{j+1}\right)\right\|_{\infty} \tag{5.14}
\end{equation*}
\]

\subsection*{5.5 Event Detection and Location}

A continuation power flow event is triggered when the value of one of the elements of an event function changes sign from one continuation step to the next. The event occurs at the point where the corresponding value of the event function passes through zero. MATPOWER provides event functions to detect the location at which the continuation power flow reaches the following:
- a specified target \(\lambda\) value
- the nose point
- the end of a full trace
- a generator reactive power limit
- a generator active power limit
- a bus voltage magnitude limit
- a branch flow limit

Each event function is registered with an event name, a flag indicating whether or not the location of the event should be pinpointed, and if so, to within what tolerance. For events that are to be located, when an event interval is detected, that is, when an element of the event function value changes sign, Matpower adjusts the continuation step size via a False Position or Regula Falsi method until it locates the point of the zero-crossing to within the specified tolerance.

The detection of an event zero, or even an event interval, can be used to trigger further actions. The CPF callback function capability was extended in Matpower 6.x to include the ability to handle events by including information about any event intervals or zeros detected. For example, Matpower includes a callback that fixes the reactive output of a generator and converts its bus from PV to PQ when the corresponding event function indicates that its reactive power limit has been reached. Another responds to the detection of the nose point by signaling the termination of the continuation. In fact, continuation power flow termination for nose point, target
lambda or full trace modes are all based on CPF callback functions in conjunction with event detection.

While MATPOWER does include a mechanism for supplying user defined callback functions, it does not yet have a corresponding mechanism for user specified event functions.

\section*{5.6 runcpf}

In Matpower, a continuation power flow is executed by calling runcpf with two Matpower cases (case structs or case file names) as the first two arguments, basecasedata and targetcasedata, respectively. The first contains the base loading/generation profile while the second contains the target loading/generation profile. In addition to printing output to the screen, which it does by default, runcpf optionally returns the solution in a results struct.
```

>> results = runcpf(basecasedata, targetcasedata);

```

Additional optional input arguments can be used to set options (mpopt) and provide file names for saving the pretty printed output (fname) or the solved case data (solvedcase).
```

>> results = runcpf(basecasedata, targetcasedata, mpopt, fname, solvedcase);

```

The results struct is a superset of the input Matpower case struct mpc, with some additional fields as well as additional columns in some of the existing data fields. In addition to the solution values included in the results for a simple power flow, shown in Table 4-1 in Section 4.4, the following additional continuation power flow solution values are stored in the cpf field as shown in Table 5-1.

The options that control the continuation power flow simulation are listed in Table 5-2. All the power flow options for Newton's method (tolerance, maximum iterations) and for controlling the output on the screen (see Tables 4-2 and 4-3) are also available with the continuation power flow.

Table 5-1: Continuation Power Flow Results
\begin{tabular}{|c|c|}
\hline name & description \\
\hline results.cpf & CPF output struct whose content depends on any user callback functions, where default contains fields: \\
\hline done_msg & string with message describing cause of continuation termination \\
\hline events(eidx) & a structure array of size \(n_{e}\), where \(n_{e}\) is the number of events located, with fields: \\
\hline k & continuation step number at which event was located \\
\hline name & name of event \\
\hline idx & index(es) of critical elements in corresponding event function, e.g. index of generator reaching VAr limit \\
\hline msg & descriptive text detailing the event \\
\hline iterations & \(n_{\text {steps }}\), number of continuation steps performed \\
\hline lam & \(1 \times n\) vector of \(\lambda\) values from correction steps \({ }^{\dagger}\) \\
\hline lam_hat & \(1 \times n\) vector of \(\lambda\) values from prediction steps \({ }^{\dagger}\) \\
\hline max_lam & maximum value of \(\lambda\) found in results.cpf.lam \\
\hline steps & \(1 \times n\) vector of step sizes for each continuation step performed \({ }^{\dagger}\) \\
\hline V & \(n_{b} \times n\) matrix of complex bus voltages from correction steps \({ }^{\dagger}\) \\
\hline V_hat & \(n_{b} \times n\) matrix of complex bus voltages from prediction steps \({ }^{\dagger}\) \\
\hline
\end{tabular}
\(\dagger n\) is one more than the number of continuation steps, i.e. \(n_{\text {steps }}+1\), so the first element corresponds to the starting point.

Table 5-2: Continuation Power Flow Options
\begin{tabular}{|c|c|c|}
\hline name & default & description \\
\hline \multirow[t]{4}{*}{cpf.parameterization} & 3 & choice of parameterization \\
\hline & & 1 - natural \\
\hline & & 2 - arc length \\
\hline & & 3 - pseudo arc length \\
\hline \multirow[t]{4}{*}{cpf.stop_at} & ' NOSE ' & determines stopping criterion \\
\hline & & ' NOSE' - stop when nose point is reached \\
\hline & & ' FULL' - trace full nose curve \\
\hline & 0 & \(\lambda_{\text {stop }}\) - stop upon reaching target \(\lambda\) value \(\lambda_{\text {stop }}\) enforce gen active power limits \\
\hline \multirow{2}{*}{cpf.enforce_p_lims} & & 0 - do not enforce limits \\
\hline & & 1 - enforce limits \\
\hline \multirow[t]{2}{*}{cpf.enforce_q_lims} & 0 & enforce gen reactive power limits at expense of \(V_{m}\) 0 - do not enforce limits \\
\hline & & 1 - enforce limits \\
\hline \multirow[t]{3}{*}{cpf.enforce_v_lims} & 0 & enforce bus voltage magnitude limits \\
\hline & & 0 - do not enforce limits \\
\hline & & 1 - enforce limits \\
\hline \multirow[t]{3}{*}{cpf.enforce_flow_lims} & 0 & enforce branch MVA flow limits \\
\hline & & 0 - do not enforce limits \\
\hline & & 1 - enforce limits \\
\hline cpf.step & 0.05 & default value for continuation power flow step size \(\sigma\) \\
\hline cpf.step_min & \(10^{-4}\) & minimum allowed step size, \(\sigma_{\text {min }}\) \\
\hline cpf.step_max & 0.2 & maximum allowed step size, \(\sigma_{\text {max }}\) \\
\hline \multirow[t]{3}{*}{cpf.adapt_step} & 0 & toggle adaptive step size feature \\
\hline & & 0 - adaptive step size disabled \\
\hline & & 1 - adaptive step size enabled \\
\hline cpf.adapt_step_damping & 0.7 & damping factor \(\beta_{\mathrm{cpf}}\) from (5.13) for adaptive step sizing \\
\hline cpf.adapt_step_tol & \(10^{-3}\) & tolerance \(\epsilon_{\text {cpf }}\) from (5.13) for adaptive step sizing \\
\hline cpf.target_lam_tol & \(10^{-5}\) & tolerance for target lambda detection \\
\hline cpf.nose_tol & \(10^{-5}\) & tolerance for nose point detection (p.u.) \\
\hline cpf.p_lims_tol & \(10^{-2}\) & tolerance for generator active power limit detection (MW) \\
\hline cpf.q_lims_tol & \(10^{-2}\) & tolerance for generator reactive power limit detection (MVAr) \\
\hline cpf.v_lims_tol & \(10^{-4}\) & tolerance for bus voltage magnitude limit detection (p.u) \\
\hline cpf.flow_lims_tol & 0.01 & tolerance for branch flow limit detection (MVA) \\
\hline \multirow[t]{5}{*}{cpf.plot.level} & 0 & control plotting of nose curve \\
\hline & & 0 - do not plot nose curve \\
\hline & & 1 - plot when completed \\
\hline & & 2 - plot incrementally at each iteration \\
\hline & & 3 - same as 2 , with pause at each iteration \\
\hline cpf.plot.bus & empty & index of bus whose voltage is to be plotted \\
\hline cpf.user_callback & empty & string or cell array of strings with names of user callback functions \({ }^{\dagger}\) \\
\hline
\end{tabular}

\footnotetext{
\({ }^{\dagger}\) See help cpf_default_callback for details.
}

\subsection*{5.6.1 CPF Callback Functions}

MATPOWER's continuation power flow provides a callback mechanism to give the user access to the iteration process for executing custom code at each iteration, for example, to implement custom incremental plotting of a PV nose curve or to handle a detected event. This callback mechanism is used internally to handle default plotting functionality as well as to handle the standard CPF events. The cpf_default_callback function, for example, is used to collect the \(\lambda\) and \(V\) results from each predictor and corrector iteration and optionally plot the PV nose curve.

The prototype for a CPF callback function is
```

function [nx, cx, done, rollback, evnts, cb_data, results] = ...
cpf_user_callback(k, nx, cx, px, done, rollback, evnts, ...
cb_data, cb_args, results)

```
and the input and output arguments are described in Tables 5-3 through 5-5 and in the help for cpf_default_callback. Each registered CPF callback function is called in three different contexts, distinguished by the value of the first argument k as follows:
1. initial - called with \(\mathrm{k}=0\), without results input/output arguments, after base power flow, before first CPF step.
2. iterations - called with \(\mathrm{k}>0\), without results input/output arguments, at each iteration, after predictor-corrector step
3. final - called with \(\mathrm{k}<0\), with results input/output arguments, after exiting predictor-corrector loop, inputs identical to last iteration call, except k negated

The user can define their own callback functions which take the same form and are called in the same contexts as cpf_default_callback. User callback functions are specified via the MATPOWER option cpf.user_callback. This option can be a string containing the name of the callback function, or a struct with the following fields, where all but the first are optional:
- \(f \mathrm{cn}\) - string with name of callback function
- priority - numerical value specifying callback priority, \({ }^{18}\) default \(=20\)
- args - arbitrary value (any type) passed to the callback as cb_args each time it is invoked

Multiple user callbacks can be registered by assigning a cell array of such strings and/or structs to the cpf.user_callback option.

\footnotetext{
\({ }^{18}\) See cpf_register_callback for details.
}

Table 5-3: Continuation Power Flow Callback Input Arguments
\begin{tabular}{|c|c|}
\hline name & description \\
\hline k & continuation step iteration count \\
\hline nx & next CPF state, corresponding to proposed next step \\
\hline cx & current CPF state, corresponding to most recent successful step \\
\hline px & previous CPF state, corresponding to last successful step prior to cx \\
\hline done & struct, with flag to indicate CPF termination and reason, with fields: \\
\hline .flag & termination flag, \(1 \rightarrow\) terminate, \(0 \rightarrow\) continue \\
\hline .msg & string containing reason for termination \\
\hline rollback & scalar flag to indicate that the current step should be rolled back and retried with a different step size, etc. \\
\hline evnts & struct array listing any events detected for this step \({ }^{\ddagger}\) \\
\hline cb_data & struct containing static data \({ }^{\S}\), with the following fields (all based on internal indexing): \\
\hline .mpc_base & Matpower case struct of base case \\
\hline .mpc_target & Matpower case struct of target case \\
\hline . Sbusb & handle of function returning \(n_{b} \times 1\) vector of complex base case bus injections in p.u. and derivatives w.r.t. \(|V|\) \\
\hline . Sbust & handle of function returning \(n_{b} \times 1\) vector of complex target case bus injections in p.u. and derivatives w.r.t. \(|V|\) \\
\hline . Ybus & bus admittance matrix \\
\hline . Yf & branch admittance matrix, "from" end of branches \\
\hline . Yt & branch admittance matrix, "to" end of branches \\
\hline .pv & list of indices of PV buses \\
\hline .pq & list of indices of PQ buses \\
\hline .ref & list of indices of reference buses \\
\hline .idx_pmax & vector of gen indices of gens at PMAX \\
\hline .mpopt & Matpower options struct \\
\hline cb_args & arbitrary data structure containing user-provided callback arguments \\
\hline results & initial value of output struct to be assigned to cpf field of results struct returned by runcpf \\
\hline
\end{tabular}
* See Table 5-5 for details of the CPF state.
\(\ddagger\) See cpf_detect_events for details of the evnts field.
§ Please note that any callback that modifies the underlying problem is responsible to update the contents of cb_data accordingly. E.g. converting a bus from PV to PQ requires updates to mpc_base, mpc_target, Sbusb, Sbust, pv, pq, and possibly ref. So, cb_data should only be thought of as static for a fixed base and target case pair.

\title{
Table 5-4: Continuation Power Flow Callback Output Arguments
}
\begin{tabular}{ll}
\hline name & description \\
\hline All are updated versions of the corresponding input arguments, see Table 5-3 for more details. \\
nx & \begin{tabular}{l} 
next CPF state, user state (cb field) should be updated here if rollback \\
is false
\end{tabular} \\
cx & \begin{tabular}{l} 
current CPF state, may contain updated this_step or this_parm values \\
to be used if rollback is true
\end{tabular} \\
done & \begin{tabular}{l} 
callback may have requested termination and set the msg field \\
callback can request a rollback step, even if it was not indicated by an \\
evnts
\end{tabular} \\
event function
\end{tabular}
* See Table 5-5 for details of the CPF state.
\(\dagger\) In this case, the callback should also modify the step size or parameterization to be used for the re-try, by setting the this_step or this_parm fields in cx.
\(\ddagger\) See cpf_detect_events for details of the evnts field.
§ It is the job of any callback modifying cb_data to ensure that all data in cb_data is kept consistent.

Table 5-5: Continuation Power Flow State
\begin{tabular}{ll}
\hline name & description \\
\hline lam_hat & value of \(\lambda\) after predictor step \\
V_hat & vector of complex bus voltages after predictor step \\
lam & value of \(\lambda\) after corrector step \\
V & vector of complex bus voltages after corrector step \\
z & normalized tangent predictor, \(\bar{z}\) \\
default_step & default step size \\
default_parm & default parameterization* \\
this_step & step size for this step only \\
this_parm & parameterization \({ }^{*}\) for this step only \\
step & current step size \\
parm & current parameterization* \\
events & event log \\
cb & user state for callbacks \({ }^{\dagger}\), a user-defined struct containing any information \\
ef & a callback function would like to pass from one invokation to the next \\
& cell array of event function values
\end{tabular}
* Corresponding to the cpf. parameterization option in Table 5-2.
\(\dagger\) Replaces cb_state from Matpower 5.

\subsection*{5.6.2 CPF Example}

The following is an example of running a continuation power flow using a version of the 9 -bus system, looking at increasing all loads by a factor of 2.5 . This example plots the nose curve shown in Figure 5-1.


Figure 5-1: Nose Curve of Voltage Magnitude at Bus 9
```

define_constants;
mpopt = mpoption('out.all', 0, 'verbose', 2);
mpopt = mpoption(mpopt, 'cpf.stop_at', 'FULL', 'cpf.step', 0.2);
mpopt = mpoption(mpopt, 'cpf.plot.level', 2);
mpcb = loadcase(t_case9_pfv2); % load base case
mpct = mpcb; % set up target case with
mpct.gen(:, [PG QG]) = mpcb.gen(:, [PG QG]) * 2.5; % increased generation
mpct.bus(:, [PD QD]) = mpcb.bus(:, [PD QD]) * 2.5; % and increased load
results = runcpf(mpcb, mpct, mpopt);

```

This should result in something like the following output to the screen.


The results of the continuation power flow are then found in the cpf field of the returned results struct.
```

>> results.cpf
ans =
V_hat: [9x17 double]
lam_hat: [1x17 double]
V: [9x17 double]
lam: [1x17 double]
steps: [1x17 double]
iterations: 16
max_lam: 0.9876
events: [1x1 struct]
done_msg: 'Traced full continuation curve in 16 continuation steps'

```

\section*{6 Optimal Power Flow}

MATPOWER includes code to solve both AC and DC versions of the optimal power flow problem. The standard version of each takes the following form:
\[
\begin{equation*}
\min _{x} f(x) \tag{6.1}
\end{equation*}
\]
subject to
\[
\begin{gather*}
g(x)=0  \tag{6.2}\\
h(x) \leq 0  \tag{6.3}\\
x_{\min } \leq x \leq x_{\max } . \tag{6.4}
\end{gather*}
\]

In both cases, the objective function \(f(x)\) consists of the polynomial cost of generator injections, the equality constraints \(g(x)\) are the power balance equations, the inequality constraints \(h(x)\) are the branch flow limits, and the \(x_{\min }\) and \(x_{\max }\) bounds include reference bus angles, voltage magnitudes (for AC ) and generator injections.

\subsection*{6.1 Standard AC OPF}

The optimization vector \(x\) for the standard AC OPF problem consists of the \(n_{b} \times 1\) vectors of voltage angles \(\Theta\) and magnitudes \(V_{m}\) and the \(n_{g} \times 1\) vectors of generator real and reactive power injections \(P_{g}\) and \(Q_{g}\).
\[
x=\left[\begin{array}{c}
\Theta  \tag{6.5}\\
V_{m} \\
P_{g} \\
Q_{g}
\end{array}\right]
\]

The objective function \(f(x)\) in (6.1) is simply a summation of individual polynomial cost functions \(f_{P}^{i}\) and \(f_{Q}^{i}\) of real and reactive power injections, respectively, for each generator:
\[
\begin{equation*}
f\left(P_{g}, Q_{g}\right)=\sum_{i=1}^{n_{g}} f_{P}^{i}\left(p_{g}^{i}\right)+f_{Q}^{i}\left(q_{g}^{i}\right) \tag{6.6}
\end{equation*}
\]

The equality constraints in (6.2) are simply the full set of \(2 \cdot n_{b}\) nonlinear real and reactive power balance equations from (4.2) and (4.3).
\[
\begin{align*}
g_{P}\left(\Theta, V_{m}, P_{g}\right) & =P_{\mathrm{bus}}\left(\Theta, V_{m}\right)+P_{d}-C_{g} P_{g}=0  \tag{6.7}\\
g_{Q}\left(\Theta, V_{m}, Q_{g}\right) & =Q_{\mathrm{bus}}\left(\Theta, V_{m}\right)+Q_{d}-C_{g} Q_{g}=0 \tag{6.8}
\end{align*}
\]

The inequality constraints (6.3) consist of two sets of \(n_{l}\) branch flow limits as nonlinear functions of the bus voltage angles and magnitudes, one for the from end and one for the to end of each branch:
\[
\begin{align*}
h_{f}\left(\Theta, V_{m}\right) & =\left|F_{f}\left(\Theta, V_{m}\right)\right|-F_{\max } \leq 0  \tag{6.9}\\
h_{t}\left(\Theta, V_{m}\right) & =\left|F_{t}\left(\Theta, V_{m}\right)\right|-F_{\max } \leq 0 . \tag{6.10}
\end{align*}
\]

The flows are typically apparent power flows expressed in MVA, but can be real power or current flows, yielding the following three possible forms for the flow constraints:
\[
F_{f}\left(\Theta, V_{m}\right)= \begin{cases}S_{f}\left(\Theta, V_{m}\right), & \text { apparent power }  \tag{6.11}\\ P_{f}\left(\Theta, V_{m}\right), & \text { real power } \\ I_{f}\left(\Theta, V_{m}\right), & \text { current }\end{cases}
\]
where \(I_{f}\) is defined in (3.9), \(S_{f}\) in (3.15), \(P_{f}=\Re\left\{S_{f}\right\}\) and the vector of flow limits \(F_{\max }\) has the appropriate units for the type of constraint. It is likewise for \(F_{t}\left(\Theta, V_{m}\right)\). The values used by Matpower's OPF for the flow limits \(F_{\text {max }}\) are specified in the RATE_A column (6) of the branch matrix, \({ }^{19}\) and the selection of flow constraint type in (6.11) is determined by the opf.flow_lim option.

The variable limits (6.4) include an equality constraint on any reference bus angle and upper and lower limits on all bus voltage magnitudes and real and reactive generator injections:
\[
\begin{align*}
\theta_{i}^{\text {ref }} \leq \theta_{i} \leq \theta_{i}^{\text {ref }}, & & i \in \mathcal{I}_{\text {ref }}  \tag{6.12}\\
v_{m}^{i, \min } \leq v_{m}^{i} \leq v_{m}^{i, \max }, & & i=1 \ldots n_{b}  \tag{6.13}\\
p_{g}^{i, \text { min }} \leq p_{g}^{i} \leq p_{g}^{i, \text { max }}, & & i=1 \ldots n_{g}  \tag{6.14}\\
q_{g}^{i, \min } \leq q_{g}^{i} \leq q_{g}^{i \text { max }}, & & i=1 \ldots n_{g} . \tag{6.15}
\end{align*}
\]

The voltage reference angle \(\theta_{i}^{\text {ref }}\) and voltage magnitude bounds \(v_{m}^{i, \text { max }}\) and \(v_{m}^{i, \min }\) are specified in columns VA (9), vMAX (12) and VMIn (13), respectively, of row \(i\) of the bus matrix. Similarly, the generator bounds \(q_{g}^{i, \text { max }}, q_{g}^{i, \text { min }}, p_{g}^{i, \text { max }}\) and \(p_{g}^{i, \text { min }}\) are specfied in columns QMAX (4), QMIN (5), PMAX (9) and PMIN (10), respectively, of row \(i\) of the gen matrix.

\subsection*{6.1.1 Cartesian vs. Polar Coordinates for Voltage}

Another variation of the standard AC OPF problem represents the bus voltages in cartesian, rather than polar, coordinates. That is, instead of \(\Theta\) and \(V_{m}\), the

\footnotetext{
\({ }^{19}\) Setting the RATE_A column (6) of branch to zero is the preferred way to indicate a completely unconstrained line.
}
optimization vector \(x\) includes the real and imaginary parts of the complex voltage, denoted respectively by \(U\) and \(W\), where \(V=U+j W\).
\[
x=\left[\begin{array}{c}
U  \tag{6.16}\\
W \\
P_{g} \\
Q_{g}
\end{array}\right]
\]

The objective function remains unchanged, but the nodal power balance constraints (6.7) and (6.8) and branch flow constraints (6.9) and (6.10) are implemented as functions of \(U\) and \(W\).
\[
\begin{align*}
g_{P}\left(U, W, P_{g}\right) & =P_{\mathrm{bus}}(U, W)+P_{d}-C_{g} P_{g}=0  \tag{6.17}\\
g_{Q}\left(U, W, Q_{g}\right) & =Q_{\mathrm{bus}}(U, W)+Q_{d}-C_{g} Q_{g}=0  \tag{6.18}\\
h_{f}(U, W) & =\left|F_{f}(U, W)\right|-F_{\max } \leq 0  \tag{6.19}\\
h_{t}(U, W) & =\left|F_{t}(U, W)\right|-F_{\max } \leq 0 . \tag{6.20}
\end{align*}
\]

In this formulation, the voltage angle reference constraint (6.12) and voltage magnitude limits (6.13) cannot be simply applied as bounds on optimization variables. These constrained quantities also become functions of \(U\) and \(W\).
\[
\begin{align*}
\theta_{i}^{\text {ref }} \leq \theta_{i}\left(u_{i}, w_{i}\right) \leq \theta_{i}^{\text {ref }}, & & i \in \mathcal{I}_{\text {ref }}  \tag{6.21}\\
v_{m}^{i, \text { min }} \leq v_{m}^{i}\left(u_{i}, w_{i}\right) \leq v_{m}^{i, \text { max }}, & & i=1 \ldots n_{b} \tag{6.22}
\end{align*}
\]

In Matpower setting the opf.v_cartesian option to 1 ( 0 by default) selects the cartesian representation for voltages when running an AC OPF. \({ }^{20}\)

\subsection*{6.1.2 Current vs. Power for Nodal Balance Constraints}

Another variation of the standard AC OPF problem uses current balance constraints in place of the power balance constraints (6.7)-(6.8) or (6.17)-(6.18). If we let \(M\) and \(N\) represent the real and imaginary parts, respectively, of the current, we can express the current balance functions for the polar form as
\[
\begin{align*}
& g_{M}\left(\Theta, V_{m}, P_{g}, Q_{g}\right)=\Re\left\{I_{\mathrm{bus}}\left(\Theta, V_{m}\right)+\left[V^{*}\right]^{-1}\left(S_{d}-C_{g} S_{g}\right)^{*}\right\}=0  \tag{6.23}\\
& g_{N}\left(\Theta, V_{m}, P_{g}, Q_{g}\right)=\Im\left\{I_{\text {bus }}\left(\Theta, V_{m}\right)+\left[V^{*}\right]^{-1}\left(S_{d}-C_{g} S_{g}\right)^{*}\right\}=0 \tag{6.24}
\end{align*}
\]

\footnotetext{
\({ }^{20}\) This option only applies to solvers based on MIPS, fmincon, IPOPT and KNITRO.
}
and for the cartesian form as
\[
\begin{align*}
g_{M}\left(U, W, P_{g}, Q_{g}\right) & =\Re\left\{I_{\text {bus }}(U, W)+\left[V^{*}\right]^{-1}\left(S_{d}-C_{g} S_{g}\right)^{*}\right\} \tag{6.25}
\end{align*}=0
\]
where \(S_{d}=P_{d}+j Q_{d}, S_{g}=P_{g}+j Q_{g}\) and \(\left[V^{*}\right]^{-1}\) is a diagonal matrix whose \(i\)-th diagonal entry is \(1 / v_{i}^{*}\), that is \(\frac{1}{v_{m}^{i}} e^{j \theta_{i}}\) or \(1 /\left(u_{i}-j w_{i}\right)\).

In this formulation, which can be selected by setting the opf.current_balance option to \(1,{ }^{21}\) the objective function and other constraints are not affected. This option can be used in conjuntion with either the polar or cartesian representation of bus voltages.

\subsection*{6.2 Standard DC OPF}

When using DC network modeling assumptions and limiting polynomial costs to second order, the standard OPF problem above can be simplified to a quadratic program, with linear constraints and a quadratic cost function. In this case, the voltage magnitudes and reactive powers are eliminated from the problem completely and real power flows are modeled as linear functions of the voltage angles. The optimization variable is
\[
x=\left[\begin{array}{c}
\Theta  \tag{6.27}\\
P_{g}
\end{array}\right]
\]
and the overall problem reduces to the following form.
\[
\begin{equation*}
\min _{\Theta, P_{g}} \sum_{i=1}^{n_{g}} f_{P}^{i}\left(p_{g}^{i}\right) \tag{6.28}
\end{equation*}
\]
subject to
\[
\begin{align*}
& g_{P}\left(\Theta, P_{g}\right)=B_{\mathrm{bus}} \Theta+P_{\mathrm{bus}, \text { shift }}+P_{d}+G_{s h}-C_{g} P_{g}=0  \tag{6.29}\\
& h_{f}(\Theta)=\quad B_{f} \Theta+P_{f, \text { shift }}-F_{\max } \leq 0  \tag{6.30}\\
& h_{t}(\Theta)=-B_{f} \Theta-P_{f, \text { shift }}-F_{\max } \leq 0  \tag{6.31}\\
& \theta_{i}^{\text {ref }} \leq \theta_{i} \leq \theta_{i}^{\text {ref }}, \quad i \in \mathcal{I}_{\text {ref }}  \tag{6.32}\\
& p_{g}^{i, \min } \leq p_{g}^{i} \leq p_{g}^{i, \text { max }}, \quad i=1 \ldots n_{g} \tag{6.33}
\end{align*}
\]

\footnotetext{
\({ }^{21}\) This option only applies to solvers based on MIPS, fmincon, IPOPT and KNITRO.
}

\subsection*{6.3 Extended OPF Formulation}

Matpower employs an extensible OPF structure to allow the user to modify or augment the problem formulation without rewriting the portions that are shared with the standard OPF formulation described above. The standard formulation is modified by introducing additional variables, user-defined costs, and/or user-defined constraints. The full extended formulation can be written as follows:
\[
\begin{equation*}
\min _{\hat{x}} f(x)+f_{u}(\hat{x}) \tag{6.34}
\end{equation*}
\]
subject to
\[
\begin{gather*}
\hat{g}(\hat{x})=0  \tag{6.35}\\
\hat{h}(\hat{x}) \leq 0  \tag{6.36}\\
\hat{x}_{\min } \leq \hat{x} \leq \hat{x}_{\max }  \tag{6.37}\\
l \leq A \hat{x} \leq u \tag{6.38}
\end{gather*}
\]

The first difference to note is that the optimization variable \(x\) from the standard OPF formulation has been augmented with additional variables \(z\) to form a new optimization variable \(\hat{x}\), and likewise with the lower and upper bounds.
\[
\hat{x}=\left[\begin{array}{c}
x  \tag{6.39}\\
z
\end{array}\right] \quad \hat{x}_{\min }=\left[\begin{array}{c}
x_{\min } \\
z_{\min }
\end{array}\right] \quad \hat{x}_{\max }=\left[\begin{array}{l}
x_{\max } \\
z_{\max }
\end{array}\right]
\]

Second, there is an additional user-defined cost term \(f_{u}(\hat{x})\) in the objective function. This cost consists of three pieces that will be described in more detail below.
\[
\begin{equation*}
f_{u}(\hat{x})=f_{\mathrm{q}}(\hat{x})+f_{\mathrm{nln}}(\hat{x})+f_{\text {legacy }}(\hat{x}) \tag{6.40}
\end{equation*}
\]

Third, the nonlinear constraints \(g\) and \(h\) are augmented with user defined additions \(g_{u}\) and \(h_{u}\) to give \(\hat{g}\) and \(\hat{h}\).
\[
\hat{g}(\hat{x})=\left[\begin{array}{c}
g(x)  \tag{6.41}\\
g_{u}(\hat{x})
\end{array}\right], \quad \hat{h}(\hat{x})=\left[\begin{array}{c}
h(x) \\
h_{u}(\hat{x})
\end{array}\right]
\]

And finally, a new set of linear constraints are included in (6.38).
Up through version 6.0 of Matpower, the OPF extensions were handled via optional input parameters that define any additional variables, \({ }^{22}\) linear constraints, \({ }^{23}\)

\footnotetext{
\({ }^{22}\) Parameters \(z_{\text {min }}\) and \(z_{\text {max }}\) in (6.39).
\({ }^{23}\) Parameters \(A, l, u\) in (6.38).
}
and costs of the pre-specified form defined by \(f_{\text {legacy }} .{ }^{24}\) This preserved the ability to use solvers that employ pre-compiled MEX files to compute all of the costs and constraints. This is referred to as Matpower's legacy extended OPF formulation [24].

For the AC OPF, subsequent versions also include the general nonlinear constraints \(g_{u}\) and \(h_{u}\), and the quadratic and general nonlinear costs \(f_{\mathrm{q}}\) and \(f_{\operatorname{mln}}\). The new quadratic cost terms can be handled by all of Matpower's AC OPF solvers, but the general nonlinear costs and constraints require a solver that uses MATLAB code to implement the function, gradient and Hessian evaluations. \({ }^{25}\)

Section 7 describes the mechanisms available to the user for taking advantage of the extensible formulation described here.

\subsection*{6.3.1 User-defined Variables}

The creation of additional user-defined \(z\) variables can be done explicitly or implicitly based on the difference between the number of columns in \(A\) and the dimension of the standard OPF optimization variable \(x\). The optional vectors \(z_{\text {min }}\) and \(z_{\max }\) are available to impose lower and upper bounds on \(z\), respectively.

\subsection*{6.3.2 User-defined Constraints}
- Linear Constraints - The user-defined linear constraints (6.38) are general linear restrictions involving all of the optimization variables and are specified via matrix \(A\) and lower and upper bound vectors \(l\) and \(u\). These parameters can be used to create equality constraints \(\left(l_{i}=u_{i}\right)\) or inequality constraints that are bounded below \(\left(u_{i}=\infty\right)\), bounded above \(\left(l_{i}=\infty\right)\) or bounded on both sides.
- Nonlinear Constraints - The user-defined general nonlinear constraints take the form
\[
\begin{array}{ll}
g_{u}^{j}(x)=0 & \forall j \in \mathcal{G}_{u} \\
h_{u}^{j}(x) \leq 0 & \forall j \in \mathcal{H}_{u} \tag{6.43}
\end{array}
\]
where \(\mathcal{G}_{u}\) and \(\mathcal{H}_{u}\) are sets of indices for user-defined equality and inequality constraint sets, respectively.
Each of these constraint sets is defined by two M-file functions, similar to those required by MIPS, one that computes the constraint values and their gradients (Jacobian), and the other that computes Hessian values.

\footnotetext{
\({ }^{24}\) Parameters \(H, C, N, \hat{r}, k, d, m\) in (6.48)-(6.51).
\({ }^{25}\) At the time of this writing, this includes solvers based on MIPS, fmincon, IpOPT and KNITRO.
}

\subsection*{6.3.3 User-defined Costs}

The user-defined cost function \(f_{u}\) consists of three terms for three different types of costs: quadratic, general nonlinear, and legacy. Each term is a simple summation over all of the cost sets of that type.
\[
\begin{align*}
f_{\mathrm{q}}(\hat{x}) & =\sum_{j} f_{\mathrm{q}}^{j}(\hat{x})  \tag{6.44}\\
f_{\operatorname{nln}}(\hat{x}) & =\sum_{j} f_{\operatorname{nln}}^{j}(\hat{x})  \tag{6.45}\\
f_{\text {legacy }}(\hat{x}) & =\sum_{j} f_{\text {legacy }}^{j}(\hat{x}) \tag{6.46}
\end{align*}
\]
- Quadratic Costs for a cost set \(j\) are specified by parameters \(Q_{j}, c_{j}\) and \(k_{j}\) that define a quadratic function of the optimization variable \(\hat{x}\).
\[
\begin{equation*}
f_{\mathrm{q}}^{j}(\hat{x})=\hat{x}^{\top} Q_{j} \hat{x}+c_{j}^{\top} \hat{x}+k_{j} \tag{6.47}
\end{equation*}
\]
- General Nonlinear Costs for a cost set \(j\) consist of a cost function \(f_{\text {nln }}^{j}(\hat{x})\) provided in the form of a function handle to an M-file function that evaluates the cost and its gradients and Hessian for a given value of \(\hat{x}\).
- Legacy Costs \(f_{\text {legacy }}^{j}(\hat{x})\) are specified in terms of parameters \(H_{j}, C_{j}, N_{j}, \hat{r}_{j}\), \(k_{j}, d_{j}\) and \(m_{j}\). For simplicity of presentation, we will drop the \(j\) subscript for the rest of this discussion. All of the parameters are \(n_{w} \times 1\) vectors except the symmetric \(n_{w} \times n_{w}\) matrix \(H\) and the \(n_{w} \times n_{\hat{x}}\) matrix \(N\). The legacy user cost function takes the form
\[
\begin{equation*}
f_{\text {legacy }}(\hat{x})=\frac{1}{2} w^{\boldsymbol{\top}} H w+C^{\boldsymbol{\top}} w \tag{6.48}
\end{equation*}
\]
where \(w\) is defined in several steps as follows. First, a new vector \(u\) is created by applying a linear transformation \(N\) and shift \(\hat{r}\) to the full set of optimization variables
\[
\begin{equation*}
u=N \hat{x}-\hat{r}, \tag{6.49}
\end{equation*}
\]
then a scaled function with a "dead zone" is applied to each element of \(u\) to produce the corresponding element of \(w\).
\[
w_{i}=\left\{\begin{array}{cc}
m_{i} f_{d_{i}}\left(u_{i}+k_{i}\right), & u_{i}<-k_{i}  \tag{6.50}\\
0, & -k_{i} \leq u_{i} \leq k_{i} \\
m_{i} f_{d_{i}}\left(u_{i}-k_{i}\right), & u_{i}>k_{i}
\end{array}\right.
\]

Here \(k_{i}\) specifies the size of the "dead zone", \(m_{i}\) is a simple scale factor and \(f_{d_{i}}\) is a pre-defined scalar function selected by the value of \(d_{i}\). Currently, Matpower implements only linear and quadratic options:
\[
f_{d_{i}}(\alpha)= \begin{cases}\alpha, & \text { if } d_{i}=1  \tag{6.51}\\ \alpha^{2}, & \text { if } d_{i}=2\end{cases}
\]
as illustrated in Figure 6-1 and Figure 6-2, respectively.


Figure 6-1: Relationship of \(w_{i}\) to \(r_{i}\) for \(d_{i}=1\) (linear option)
This form for \(f_{\text {legacy }}\) provides the flexibility to handle a wide range of costs, from simple linear functions of the optimization variables to scaled quadratic penalties on quantities, such as voltages, lying outside a desired range, to functions of linear combinations of variables, inspired by the requirements of price coordination terms found in the decomposition of large loosely coupled problems encountered in our own research.
Some limitations are imposed on the parameters in the case of the DC OPF since Matpower uses a generic quadratic programming (QP) solver for the optimization. In particular, \(k_{i}=0\) and \(d_{i}=1\) for all \(i\), so the "dead zone" is not considered and only the linear option is available for \(f_{d_{i}}\). As a result, for the DC case (6.50) simplifies to \(w_{i}=m_{i} u_{i}\).


Figure 6-2: Relationship of \(w_{i}\) to \(r_{i}\) for \(d_{i}=2\) (quadratic option)

\subsection*{6.4 Standard Extensions}

In addition to making this extensible OPF structure available to end users, MatPOWER also takes advantage of it internally to implement several additional capabilities.

\subsection*{6.4.1 Piecewise Linear Costs}

The standard OPF formulation in (6.1)-(6.4) does not directly handle the nonsmooth piecewise linear cost functions that typically arise from discrete bids and offers in electricity markets. When such cost functions are convex, however, they can be modeled using a constrained cost variable (CCV) method. The piecewise linear cost function \(c(x)\) is replaced by a helper variable \(y\) and a set of linear constraints that form a convex "basin" requiring the cost variable \(y\) to lie in the epigraph of the function \(c(x)\).

Figure 6-3 illustrates a convex \(n\)-segment piecewise linear cost function
\[
c(x)=\left\{\begin{array}{cc}
m_{1}\left(x-x_{1}\right)+c_{1}, & x \leq x_{1}  \tag{6.52}\\
m_{2}\left(x-x_{2}\right)+c_{2}, & x_{1}<x \leq x_{2} \\
\vdots & \vdots \\
m_{n}\left(x-x_{n}\right)+c_{n}, & x_{n-1}<x
\end{array}\right.
\]
defined by a sequence of points \(\left(x_{j}, c_{j}\right), j=0 \ldots n\), where \(m_{j}\) denotes the slope of


Figure 6-3: Constrained Cost Variable
the \(j\)-th segment
\[
\begin{equation*}
m_{j}=\frac{c_{j}-c_{j-1}}{x_{j}-x_{j-1}}, \quad j=1 \ldots n \tag{6.53}
\end{equation*}
\]
and \(x_{0}<x_{1}<\cdots<x_{n}\) and \(m_{1} \leq m_{2} \leq \cdots<m_{n}\).
The "basin" corresponding to this cost function is formed by the following \(n\) constraints on the helper cost variable \(y\) :
\[
\begin{equation*}
y \geq m_{j}\left(x-x_{j}\right)+c_{j}, \quad j=1 \ldots n \tag{6.54}
\end{equation*}
\]

The cost term added to the objective function in place of \(c(x)\) is simply the variable \(y\).
Matpower uses this CCV approach internally to automatically generate the appropriate helper variable, cost term and corresponding set of constraints for any piecewise linear costs on real or reactive generation. All of Matpower's OPF solvers, for both AC and DC OPF problems, use the CCV approach with the exception of two that are part of the optional TSPOPF package [25], namely the step-controlled primal/dual interior point method (SCPDIPM) and the trust region based augmented Lagrangian method (TRALM), both of which use a cost smoothing technique instead [26].

Note that MATPOWER (in opf_setup) automatically converts any single-segment piecewise linear costs into polynomial (linear) form to avoid unnecessarily creating extra variables and constraints. It is this modified cost, rather than the original piecewise linear equivalent, that is returned in the gencost field of the results struct.

\subsection*{6.4.2 Dispatchable Loads}

A simple approach to dispatchable or price-sensitive loads is to model them as negative real power injections with associated negative costs. This is done by specifying a generator with a negative output, ranging from a minimum injection equal to the negative of the largest possible load to a maximum injection of zero.

Consider the example of a price-sensitive load whose marginal benefit function is shown in Figure 6-4. The demand \(p_{d}\) of this load will be zero for prices above \(\lambda_{1}, p_{1}\) for prices between \(\lambda_{1}\) and \(\lambda_{2}\), and \(p_{1}+p_{2}\) for prices below \(\lambda_{2}\).


Figure 6-4: Marginal Benefit or Bid Function

This corresponds to a negative generator with the piecewise linear cost curve shown in Figure 6-5. Note that this approach assumes that the demand blocks can be partially dispatched or "split". Requiring blocks to be accepted or rejected in their entirety would pose a mixed-integer problem that is beyond the scope of the current MATPOWER implementation.

It should be noted that, with this definition of dispatchable loads as negative generators, if the negative cost corresponds to a benefit for consumption, minimizing the cost \(f(x)\) of generation is equivalent to maximizing social welfare.

With an AC network model, there is also the question of reactive dispatch for such loads. Typically the reactive injection for a generator is allowed to take on any value within its defined limits. Since this is not normal load behavior, the model used


Figure 6-5: Total Cost Function for Negative Injection
in MATPOWER assumes that dispatchable loads maintain a constant power factor. When formulating the AC OPF problem, MatPower will automatically generate an additional equality constraint to enforce a constant power factor for any "negative generator" being used to model a dispatchable load.

The power factor, which can be lagging or leading, is determined by the ratio of reactive to active power for the load and is specified by the active and reactive limits defining the nominal load in the gen matrix. For all dispatchable loads, the value in the PMIN column is negative and defines the nominal active power of the load. And PMAX is zero, allowing the load to be fully curtailed depending on the price. The reactive limits, QMIN and QMAX, depend on whether the power flow is lagging or leading. One defines the nominal reactive load and the other must be zero to allow the load to be fully curtailed. The values of PG and QG must be defined to be consistent with the nominal power factor.
- Lagging Power Factor - The reactive injection is negative, meaning that reactive power is consumed by the load. Hence, QMIN is negative, QMAX is zero, and PG and QG must be set so that QG is equal to PG * QMIN/PMIn.
- Leading Power Factor - The reactive injection is positive, that is, reactive power is produced by the load. Hence, QMAX is positive, QMIN is zero, and PG and QG must be set so that QG is equal to PG * QMAX/PMIN.

\subsection*{6.4.3 Generator Capability Curves}

The typical AC OPF formulation includes box constraints on a generator's real and reactive injections, specified as simple lower and upper bounds on \(p\) ( \(p_{\min }\) and \(p_{\max }\) ) and \(q\left(q_{\min }\right.\) and \(\left.q_{\max }\right)\). On the other hand, the true \(P-Q\) capability curves of physical generators usually involve some tradeoff between real and reactive capability, so that it is not possible to produce the maximum real output and the maximum (or minimum) reactive output simultaneously. To approximate this tradeoff, MatPOWER includes the ability to add an upper and lower sloped portion to the standard box constraints as illustrated in Figure 6-6, where the shaded portion represents the feasible operating region for the unit.


Figure 6-6: Generator \(P-Q\) Capability Curve

The two sloped portions are constructed from the lines passing through the two pairs of points defined by the six parameters \(p_{1}, q_{1}^{\min }, q_{1}^{\max }, p_{2}, q_{2}^{\min }\), and \(q_{2}^{\max }\). If these six parameters are specified for a given generator in columns PC1-QC2MAX (11-16), MATPOWER automatically constructs the corresponding additional linear inequality constraints on \(p\) and \(q\) for that unit.

If one of the sloped portions of the capability constraints is binding for generator \(k\), the corresponding shadow price is decomposed into the corresponding \(\mu_{P_{\max }}\) and \(\mu_{Q_{\min }}\) or \(\mu_{Q_{\max }}\) components and added to the respective column (MU_PMAX, MU_QMIN or MU_QMAX) in the \(k^{\text {th }}\) row of gen.

\subsection*{6.4.4 Branch Angle Difference Limits}

The difference between the bus voltage angle \(\theta_{f}\) at the from end of a branch and the angle \(\theta_{t}\) at the to end can be bounded above and below to act as a proxy for a transient stability limit, for example. If these limits are provided in columns ANGMIN (12) and ANGMAX (13) of the branch matrix, MATPOWER creates the corresponding constraints on the voltage angle variables. \({ }^{26}\)

\subsection*{6.5 Solvers}

Early versions of Matpower relied on Matlab's Optimization Toolbox [27] to provide the NLP and QP solvers needed to solve the AC and DC OPF problems, respectively. While they worked reasonably well for very small systems, they did not scale well to larger networks. Eventually, optional packages with additional solvers were added to improve performance, typically relying on MATLAB extension (MEX) files implemented in Fortran or C and pre-compiled for each machine architecture. Some of these MEX files are distributed as optional packages due to differences in terms of use. For DC optimal power flow, there is a MEX build [28] of the high performance interior point BPMPD solver [29] for LP/QP problems. For the AC OPF problem, the MINOPF [30] and TSPOPF [25] packages provide solvers suitable for much larger systems. The former is based on MINOS [31] and the latter includes the primal-dual interior point and trust region based augmented Lagrangian methods described in [26]. Matpower version 4 and later also includes the option to use the open-source IPOPT solver \({ }^{27}\) for solving both AC and DC OPFs, based on the Matlab MEX interface to \(\mathrm{IPOPT}^{28}\). It also includes the option to use CPLEX \({ }^{29}\) or MOSEK \({ }^{30}\) for DC OPFs. Matpower 4.1 added the option to use KNITRO [32] \({ }^{31}\)

\footnotetext{
\({ }^{26}\) The voltage angle difference for branch \(k\) is taken to be unbounded below if branch ( k , ANGMIN) is less than or equal to -360 and unbounded above if branch ( \(k\), ANGMAX) is greater than or equal to 360 . If both parameters are zero, the voltage angle difference is unconstrained.
\({ }^{27}\) Available from http://www.coin-or.org/projects/Ipopt.xml.
\({ }^{28}\) See https://projects.coin-or.org/Ipopt/wiki/MatlabInterface.
\({ }^{29}\) See http://www.ibm.com/software/integration/optimization/cplex-optimizer/.
\({ }^{30}\) See http://www.mosek.com/.
\({ }^{31}\) See http://www.ziena.com/.
}
for AC OPFs and the Gurobi Optimizer [33] \({ }^{32}\) for DC OPFs and Matpower 5 added GLPK [34] and 5.1 added CLP [35]. See Appendix G for more details on these optional packages.

Beginnning with version 4, Matpower also includes its own primal-dual interior point method implemented in pure-MatLaB code, derived from the MEX implementation of the algorithms described in [26]. This solver is called MIPS (Matpower Interior Point Solver) and is described in more detail in Appendix A. If no optional packages are installed, MIPS will be used by default for both the AC OPF and as the QP solver used by the DC OPF. The AC OPF solver also employs a unique technique for efficiently forming the required Hessians via a few simple matrix operations [36-38]. This solver has application to general nonlinear optimization problems outside of Matpower and can be called directly as mips. There is also a convenience wrapper function called qps_mips making it trivial to set up and solve LP and QP problems, with an interface similar to quadprog from the Matlab Optimization Toolbox.

\section*{6.6 runopf}

In Matpower, an optimal power flow is executed by calling runopf with a case struct or case file name as the first argument (casedata). In addition to printing output to the screen, which it does by default, runopf optionally returns the solution in a results struct.
```

>> results = runopf(casedata);

```

The results struct is a superset of the input Matpower case struct mpc, with some additional fields as well as additional columns in some of the existing data fields. In addition to the solution values included in the results for a simple power flow, shown in Table 4-1 in Section 4.4, the following additional optimal power flow solution values are stored as shown in Table 6-1.

\footnotetext{
\({ }^{32}\) See http://www.gurobi.com/.
}

Table 6-1: Optimal Power Flow Results
\begin{tabular}{ll}
\hline name & description \\
\hline results.f & final objective function value \\
results.x & final value of optimization variables (internal order) \\
results.om & OPF model object \({ }^{\dagger}\) \\
results.bus(:, LAM_P) & Lagrange multiplier on real power mismatch \\
results.bus(:, LAM_Q) & Lagrange multiplier on reactive power mismatch \\
results.bus(:, MU_VMAX) & Kuhn-Tucker multiplier on upper voltage limit \\
results.bus(:, MU_VMIN) & Kuhn-Tucker multiplier on lower voltage limit \\
results.gen(:, MU_PMAX) & Kuhn-Tucker multiplier on upper \(P_{g}\) limit \\
results.gen(:, MU_PMIN) & Kuhn-Tucker multiplier on lower \(P_{g}\) limit \\
results.gen(:, MU_QMAX) & Kuhn-Tucker multiplier on upper \(Q_{g}\) limit \\
results.gen(:, MU_QMIN) & Kuhn-Tucker multiplier on lower \(Q_{g}\) limit \\
results.branch(:, MU_SF) & Kuhn-Tucker multiplier on flow limit at "from" bus \\
results.branch(:, MU_ST) & Kuhn-Tucker multiplier on flow limit at "to" bus \\
results.mu & shadow prices of constraints \({ }^{\ddagger}\) \\
results.g & (optional) constraint values \\
results.dg & (optional) constraint 1 st derivatives \\
results.raw & raw solver output in form returned by MINOS, and more \({ }^{\ddagger}\) \\
results.var.val & final value of optimization variables, by named subset \({ }^{\ddagger}\) \\
results.var.mu & shadow prices on variable bounds, by named subset \({ }^{\ddagger}\) \\
results.nle & shadow prices on nonlinear equality constraints, by named subset \({ }^{\ddagger}\) \\
results.nli & shadow prices on nonlinear inequality constraints, by named subset \({ }^{\ddagger}\) \\
results.lin & shadow prices on linear constraints, by named subset \(\ddagger\) \\
results.cost & final value of user-defined costs, by named subset \({ }^{\ddagger}\) \\
\hline
\end{tabular}

\footnotetext{
\(\dagger\) See help for opf_model and opt_model for more details.
\(\ddagger\) See help for opf for more details.
}

Additional optional input arguments can be used to set options (mpopt) and provide file names for saving the pretty printed output (fname) or the solved case data (solvedcase).
```

>> results = runopf(casedata, mpopt, fname, solvedcase);

```

Some of the main options that control the optimal power flow simulation are listed in Tables 6-2 and 6-3. There are many other options that can be used to control the termination criteria and other behavior of the individual solvers. See Appendix C or the mpoption help for details. As with runpf the output printed to the screen can be controlled by the options in Table 4-3, but there are additional output options for the OPF, related to the display of binding constraints that are listed Table 6-4, along with an option that can be used to force the AC OPF to return information about the constraint values and Jacobian and the objective function gradient and Hessian.

For OPF problems, the preferred way to eliminate the flow limit on a branch is to set the RATE A column (6) of the corresponding row in the branch matrix to zero. This indicates a completely unconstrained flow (as opposed to zero flow).

By default, runopf solves an AC optimal power flow problem using a primal dual interior point method. To run a DC OPF, the model option must be set to 'DC'. For convenience, MATPOWER provides a function rundcopf which is simply a wrapper that sets the model to ' DC ' before calling runopf.

Internally, the runopf function does a number of conversions to the problem data before calling the appropriate solver routine for the selected OPF algorithm. This external-to-internal format conversion is performed by the ext2int function, described in more detail in Section 7.3.1, and includes the elimination of out-of-service equipment, the consecutive renumbering of buses and the reordering of generators by increasing bus number. All computations are done using this internal indexing. When the simulation has completed, the data is converted back to external format by int2ext before the results are printed and returned. In addition, both ext2int and int2ext can be customized via user-supplied callback routines to convert data needed by user-supplied variables, constraints or costs into internal indexing.

Table 6-2: Optimal Power Flow Solver Options
\begin{tabular}{|c|c|c|c|}
\hline name & default & description & \\
\hline \multirow[t]{9}{*}{opf.ac.solver} & \multirow[t]{9}{*}{'DEFAULT'} & AC optimal & power flow solver: \\
\hline & & 'DEFAULT' & - choose default solver, i.e. 'MIPS' \\
\hline & & 'MIPS' & - MIPS, Matpower Interior Point Solver, primal/dual interior point method \({ }^{\dagger}\) \\
\hline & & \begin{tabular}{l}
'FMINCON' \\
'IPOPT'
\end{tabular} & - Matlab Optimization Toolbox, fmincon - Ipopt* \\
\hline & & 'KNITRO' & - KNITRO* \\
\hline & & 'MINOPF' & - MINOPF*, MINOS-based solver \\
\hline & & 'PDIPM ' & - PDIPM, primal/dual interior point method \({ }^{\ddagger}\) \\
\hline & & 'SDPOPF' & - SDPOPF, solver based on semidefinite relaxation \\
\hline & & 'TRALM \({ }^{\prime}\) & - TRALM, trust region based augmented Langrangian method \\
\hline \multirow[t]{10}{*}{opf.dc.solver} & \multirow[t]{10}{*}{'DEFAULT'} & \multicolumn{2}{|l|}{\begin{tabular}{l}
DC optimal power flow solver: \\
'DEFAULT' - choose default solver based on availability in the following order: 'GUROBI', 'CPLEX', \\
'MOSEK', 'OT', 'GLPK' (linear costs only), \\
'BPMPD', 'MIPS'
\end{tabular}} \\
\hline & & 'MIPS' & - MIPS, Matpower Interior Point Solver, primal/dual interior point method \({ }^{\dagger}\) \\
\hline & & 'BPMPD ' & - BPMPD* \\
\hline & & ' CLP' & - CLP* \\
\hline & & ' CPLEX ' & - CPLEX* \\
\hline & & ' GLPK ' & - GLPK (no quadratic costs) \\
\hline & & 'GUROBI' & - Gurobi* \({ }^{*}\) \\
\hline & & 'IPOPT' & - IPOPT* \\
\hline & & 'MOSEK ' & - MOSEK* \\
\hline & & 'OT' & - Matlab Opt Toolbox, quadprog, linprog \\
\hline
\end{tabular}

\footnotetext{
* Requires the installation of an optional package. See Appendix G for details on the corresponding package.
\(\dagger\) For MIPS-sc, the step-controlled version of this solver, the mips.step_control option must be set to 1 .
\(\ddagger\) For SC-PDIPM, the step-controlled version of this solver, the pdipm.step_control option must be set to 1 .
}

Table 6-3: Other OPF Options
\begin{tabular}{|c|c|c|}
\hline name & default & description \\
\hline opf.current_balance & 0 & use current, as opposed to power, balance formulation for AC OPF, 0 or 1 \\
\hline opf.v_cartesian & 0 & use cartesian, as opposed to polar, representation for voltages for AC OPF, 0 or 1 \\
\hline opf.violation & \(5 \times 10^{-6}\) & constraint violation tolerance \\
\hline opf.use_vg & 0 & \begin{tabular}{l}
respect generator voltage setpoint, 0 or \(1^{\dagger}\) \\
0 - use voltage magnitude limits specified in bus, ignore VG in gen \\
1 - replace voltage magnitude limits specified in bus by VG in corresponding gen
\end{tabular} \\
\hline opf.flow_lim & 'S' & \begin{tabular}{l}
quantity to limit for branch flow constraints \\
'S' - apparent power flow (limit in MVA) \\
' \(\mathrm{P}^{\prime}\) - active power flow (limit in MW) \\
'I' - current magnitude (limit in MVA at 1 p.u. voltage) \\
'2' - same as 'P', but implemented using square of active flow, rather than simple max
\end{tabular} \\
\hline opf.ignore_angle_lim & 0 & ignore angle difference limits for branches 0 - include angle difference limits, if specified 1 - ignore angle difference limits even if specified \\
\hline opf.softlims.default & 1 & \begin{tabular}{l}
behavior of OPF soft limits for which parameters are not explicitly provided \\
0 - do not include softlims if not explicitly specified \\
1 - include softlims with default values if not explicitly specified
\end{tabular} \\
\hline opf.init_from_mpc* & -1 & \begin{tabular}{l}
specify whether to use the current state in Matpower case to initialize OPF \\
-1 - MATPOWER decides based on solver/algorithm \\
0 - ignore current state in MATPOWER case \({ }^{\ddagger}\) \\
1 - use current state in Matpower case
\end{tabular} \\
\hline opf.start & 0 & \begin{tabular}{l}
strategy for initializing OPF starting point \\
0 - default, Matpower decides based on solver, (currently identical to 1) \\
1 - ignore current state in Matpower case \({ }^{\ddagger}\) \\
2 - use current state in Matpower case \\
3 - solve power flow and use resulting state
\end{tabular} \\
\hline opf.return_raw_der & 0 & for AC OPF, return constraint and derivative info in results.raw (in fields \(\mathrm{g}, \mathrm{dg}, \mathrm{df}, \mathrm{d} 2 \mathrm{f}\) ) \\
\hline
\end{tabular}

\footnotetext{
* Deprecated. Use opf.start instead.
\(\dagger\) Using a value between 0 and 1 results in the limits being determined by the corresponding weighted average of the 2 options.
\(\ddagger\) Only applies to fmincon, Ipopt, KNITRO and MIPS solvers, which use an interior point estimate; others use current state in Matpower case, as with opf.start \(=2\).
}

Table 6-4: OPF Output Options
\begin{tabular}{|c|c|c|}
\hline name & default & description \\
\hline \multirow[t]{5}{*}{out.lim.all} & \multirow[t]{5}{*}{-1} & controls constraint info output \\
\hline & & -1 - individual flags control what is printed \\
\hline & & \(0-\) do not print any constraint info \({ }^{\dagger}\) \\
\hline & & 1 - print only binding constraint info \({ }^{\dagger}\) \\
\hline & & 2 - print all constraint info \({ }^{\dagger}\) \\
\hline \multirow[t]{4}{*}{out.lim.v} & \multirow[t]{4}{*}{1} & control output of voltage limit info \\
\hline & & \(0-\) do not print \\
\hline & & 1 - print binding constraints only \\
\hline & & 2 - print all constraints \\
\hline out.lim.line & 1 & control output of line flow limit info \({ }^{\ddagger}\) \\
\hline out.lim.pg & 1 & control output of gen active power limit info \({ }^{\ddagger}\) \\
\hline out.lim.qg & 1 & control output of gen reactive power limit info \({ }^{\ddagger}\) \\
\hline
\end{tabular}

\footnotetext{
\(\dagger\) Overrides individual flags.
\(\ddagger\) Takes values of 0,1 or 2 as for out.lim.v.
}

\section*{7 Extending the OPF}

The extended OPF formulation described in Section 6.3 allows the user to modify the standard OPF formulation to include additional variables, costs and/or constraints. There are two primary mechanisms available for the user to accomplish this. The first is by directly constructing the full parameters for the addional costs or constraints and supplying them either as fields in the case struct or directly as arguments to the opf function. The second, and more powerful, method is via a set of callback functions that customize the OPF at various stages of the execution. Matpower includes several examples of using the latter method, for example to add a fixed zonal reserve requirement, to implement interface flow limits, dispatchable DC transmission lines, or branch flow soft limits.

\subsection*{7.1 Direct Specification}

This section describes the additional fields that can be added to mpc (the Matpower case struct) or, alternatively, the input parameters that can be passed directly to the opf function. The former is the preferred approach, as it allows the normal use of runopf and related functions.

\subsection*{7.1.1 User-defined Variables}

In the case of direct specification, additional variables are created implicitly based on the difference between the number of columns in \(A\) and the number \(n_{x}\) of standard OPF variables. If \(A\) has more columns than \(x\) has elements, the extra columns are assumed to correspond to a new \(z\) variable. The initial value and lower and upper bounds for \(z\) can also be specified in the optional fields or arguments, \(\mathrm{zO}, \mathrm{zl}\) and zu , respectively.

\subsection*{7.1.2 User-defined Constraints}
- Linear Constraints can be added by specifying the \(A, l\) and \(u\) parameters of (6.38) directly as fields or arguments of the same names, A, 1 and \(u\), respectively, where A is sparse.
- Nonlinear Constraints for a constraint set \(j\), that is, \(g_{u}^{j}(x)\) or \(h_{u}^{j}(x)\) from (6.42) or (6.43), are implemented by defining two M-file functions, similar to
those required by MIPS. The first is a function to compute the constraint values and their gradients (Jacobian), \({ }^{33}\) with calling syntax:
```

[g, dg] = my_constraint_fcn(x, <p1>, <p2>, ...)

```

Here <p1> and <p2> represent arbitrary optional input parameters that remain constant for all calls to the function. The second is a function to compute the Hessian of the corresponding term in the Lagrangian function, that is the term corresponding to \(\lambda^{\top} g_{u}^{j}(x)\) or \(\mu^{\top} h_{u}^{j}(x)\). This function has the following calling syntax:
d2G = my_constraint_hess(x, lambda, <p1>, <p2>, ...)
Once again this is similar to the form of the Hessian evaluation function expected by MIPS. \({ }^{34}\)
In order to add a set of user-defined nonlinear constraints to the formulation through direct specification, an entry must be added to the optional user_constraints.nle (nonlinear equality) or user_constraints.nli (nonlinear inequality) fields of the Matpower case struct. These fields are cell arrays, in which each element (also a cell array) specifies a constraint set of the corresponding kind. The format is as follows, where the details are described in Table 7-1.
```

mpc.user_constraints.nle = {
{name, N, g_fcn, hess_fcn, varsets, params},
};

```

There is an example in the OPF testing code (e.g. t_opf_mips) near the end, that implements a nonlinear relationship between three different generator outputs by adding the following line.

\footnotetext{
\({ }^{33}\) The differences between this function and the one required by MIPS is that (1) this one represents a single set of equality constraints or inequality constraints, not both, (2) it returns in dg the \(m \times n\) Jacobian (for \(m\) constraints and \(n\) variables), whereas MIPS expects the \(n \times m\) transpose, (3) this function allows arbitrary additional parameters, and (4) x can be a cell array of sub-vectors of optimization variable \(\hat{x}\) (see varsets in Table 7-1).
\({ }^{34}\) The differences between this function and the one required by MIPS is that (1) this one returns the Hessian of a single term of the Lagrangian, not of the full Lagrangian, (2) the lambda argument is a simple vector of multipliers ( \(\lambda\) or \(\mu\) ) corresponding just to this set of constraints, not a struct containing the full \(\lambda\) and \(\mu\) vectors, in fields eqnonlin and ineqnonlin, respectively, and (3) this function allows arbitrary additional parameters, and (4) x can be a cell array of sub-vectors of optimization variable \(\hat{x}\) (see varsets in Table 7-1).
}

Table 7-1: User-defined Nonlinear Constraint Specification
\begin{tabular}{ll}
\hline name & description \\
\hline name & string with name of constraint set, used to label multipliers in results struct \\
N & number of constraints, i.e. dimension of \(g_{u}^{j}(x)\left(\right.\) or \(h_{u}^{j}(x)\) as the case may be) \\
g_fcn & \begin{tabular}{l} 
string containing name of function to evaluate constraint and gradients (Jacobian) \\
hess_fcn \\
string containing name of function to evaluate Hessian of corresponding term of \\
varsets
\end{tabular} \begin{tabular}{l} 
Lagrangian \\
cell array of variable set names, specifying sub-vectors of optimization vector \(\hat{x}\) to \\
be passed as inputs in \(\mathrm{x}^{\dagger}\)
\end{tabular} \\
params & \begin{tabular}{l} 
cell array of optional, arbitrary parameters to pass to each call to the constraint and \\
Hessian evaluation functions
\end{tabular} \\
\hline
\end{tabular}
\(\dagger\) If varsets is empty, x will be the full optimization vector \(\hat{x}\), otherwise it will be a cell array of sub-vectors of \(\hat{x}\) for the specified variable sets. Valid names include 'Va', 'Vm', 'Pg', and ' Qg '. It can include others depending on the OPF extensions in use. See the variable names displayed by results.om for a complete list for your problem.
```

mpc.user_constraints.nle = {
{'Pg_usr', 1, 'opf_nle_fcn1', 'opf_nle_hess1', {'Pg'}, {}}
};

```

This adds a single constraint named 'Pg_usr' as a function of the vector of generator active injections ( x will be \(\{\mathrm{Pg}\}\), where Pg is a sub-vector of the optimization vector \(\hat{x}\) containing only the generator active injections). The constraints and gradients are evaluated by a function named 'opf_nle_fcn1' and the Hessian of the corresponding term of the Lagrangian by a function named 'opf nle_hess1', neither of which expect any additional parameters.

\subsection*{7.1.3 User-defined Costs}
- Quadratic Costs by direct specification are not currently supported.
- General Nonlinear Costs by direct specification are not currently supported.
- Legacy Costs - To add legacy costs directly, the parameters \(H, C, N, \hat{r}, k\), \(d\) and \(m\) of (6.48)-(6.51) described in Section 6.3.3 are specified as fields or arguments \(\mathrm{H}, \mathrm{Cw}, \mathrm{N}\) and fparm, respectively, where fparm is the \(n_{w} \times 4\) matrix
\[
f_{\mathrm{parm}}=\left[\begin{array}{llll}
d & \hat{r} & k & m \tag{7.1}
\end{array}\right] .
\]

When specifying additional costs, N and Cw are required, while H and fparm are optional. The default value for \(H\) is a zero matrix, and the default for \(f_{\text {parm }}\) is
such that \(d\) and \(m\) are all ones and \(\hat{r}\) and \(k\) are all zeros, resulting in simple linear cost, with no shift or "dead-zone". N and H should be specified as sparse matrices.

\subsection*{7.1.4 Additional Comments}

For a simple formulation extension to be used for a small number of OPF cases, this method has the advantage of being direct and straightforward. While Matpower does include code to eliminate the columns of \(A\) and \(N\) corresponding to \(V_{m}\) and \(Q_{g}\) when running a \(\mathrm{DC} \mathrm{OPF}^{35}\), as well as code to reorder and eliminate columns appropriately when converting from external to internal data formats, this mechanism still requires the user to take special care in preparing the \(A\) and \(N\) matrices to ensure that the columns match the ordering of the elements of the optimization vectors \(x\) and \(z\). All extra constraints and variables must be incorporated into a single set of parameters that are constructed before calling the OPF. The bookkeeping needed to access the resulting variables and shadow prices on constraints and variable bounds must be handled manually by the user outside of the OPF, along with any processing of additional input data and processing, printing or saving of the additional result data. Making further modifications to a formulation that already includes user-supplied costs, constraints or variables, requires that both sets be incorporated into a new single consistent set of parameters.

\subsection*{7.2 Callback Functions}

The second method, based on defining a set of callback functions, offers several distinct advantages, especially for more complex scenarios or for adding a feature for others to use, such as the zonal reserve requirement or the interface flow limits mentioned previously. This approach makes it possible to:
- define and access variable/constraint/cost sets as individual named blocks
- define constraints, costs only in terms of variables directly involved
- pre-process input data and/or post-process result data
- print and save new result data
- simultaneously use multiple, independently developed extensions (e.g. zonal reserve requirements and interface flow limits)

\footnotetext{
\({ }^{35}\) Only if they contain all zeros.
}

With this approach the OPF formulation is modified in the formulation callback, which is described and illustrated below in Section 7.3.2. The modifications to the formulation are handled by adding variables, costs and/or constraints to the OPF Model object (om) using one of the add_* methods. Please see the documentation for each of these methods for more details.

\subsection*{7.2.1 User-defined Variables}

Additional variables are added using the add_var method.
```

om.add_var('myV', N, myV0, myV_min, myV_max);

```

Here myv is the name of the variable set, N the number of variables being added (i.e. this is an \(N \times 1\) vector being appended to the current optimization variable \(\hat{x}\) ), and \(m y V 0, m y V \_m i n, m y V \_m a x ~ a r e ~ v e c t o r s ~ o f ~ i n i t i a l ~ v a l u e s, ~ l o w e r ~ b o u n d s ~ a n d ~ u p p e r ~ b o u n d s, ~\) respectively.

\subsection*{7.2.2 User-defined Costs}
- Quadratic Costs are added using the add_quad_cost method.
```

om.add_quad_cost('myQcost', Q, c, k, varsets);

```

Here myQcost is the name of the cost set, \(\mathrm{Q}, \mathrm{c}\) and k are the \(Q_{j}, c_{j}\) and \(k_{j}\) parameters from (6.47), respectively, and var_sets is an optional cell array of variable set names. If var_sets is provided, the dimensions of \(Q\) and \(c\) will correspond, not to the full optimization vector \(\hat{x}\), but to the subvector formed by stacking only the variable sets specified in var_sets.
- General Nonlinear Costs are added using the add_nln_cost method.
```

om.add_nln_cost('myNonlinCost', 1, fcn, varsets);

```

Here myNonlinCost is the name of the cost set, fcn is a function handle and var_sets is an optional cell array of variable set names. The function handle fcn must point to a function that implements the \(f_{\text {nln }}^{j}(\hat{x})\) function from (6.46), returning the value of the function, along with its gradient and Hessian. This function has a format similar to that required by MIPS for its cost function.

If var_sets is provided, instead of the full optimization vector \(\hat{x}\), the x passed to the function f cn will be a cell array of sub-vectors of \(\hat{x}\) corresponding to the specified variable sets.
- Legacy Costs are added using the add_legacy_cost method.
```

om.add_legacy_cost('myLegacyCost', cp, varsets);

```

Here myLegacyCost is the name of the cost set, cp is a struct containing the cost parameters \(H, C, N, \hat{r}, k, d\) and \(m\) from (6.48)-(6.51) in fields \(\mathrm{H}, \mathrm{Cw}, \mathrm{N}\), \(\mathrm{rh}, \mathrm{kk}\), dd and mm , respectively, and var_sets is an optional cell array of variable set names. If var_sets is provided, the number of columns in N will correspond, not to the full optimization vector \(\hat{x}\), but to the subvector formed by stacking only the variable sets specified in var_sets.

\subsection*{7.2.3 User-defined Constraints}
- Linear Constraints are added using the add_lin_constraint method.
```

om.lin_constraint('myLinCons', A, l, u, varsets);

```

Here myLinCons is the name of the constraint set, \(\mathrm{A}, \mathrm{l}\) and u correspond to additional rows to add to the \(A, l\) and \(u\) parameters in (6.38), respectively, and var_sets is an optional cell array of variable set names. If var_sets is provided, the number of columns in A will correspond, not to the full optimization vector \(\hat{x}\), but to the subvector formed by stacking only the variable sets specified in var_sets.
- Nonlinear Constraints are added using the add_nln_constraint method.
```

om.nln_constraint('myNonlinCons', N, iseq, fcn, hess, varsets);

```

Here myNonlinCons is the name of the constraint set, N is the number of constraints being added, iseq is a true or false value indicating whether this is a set of equality (or else inequality) constraints, fcn and hess are function handles and var_sets is an optional cell array of variable set names. The function handle fen must point to a function that implements the corresponding constraint function \(g_{u}^{j}(x)\) or \(h_{u}^{j}(x)\) and its gradients, and hess to one that evaluates the corresponding term in the Hessian of the Lagrangian function. This function has a format similar to that required by MIPS for its cost function. See more details in Section 7.1.2.

If var_sets is provided, instead of the full optimization vector \(\hat{x}\), the x passed to the function fcn will be a cell array of sub-vectors of \(\hat{x}\) corresponding to the specified variable sets.

\subsection*{7.3 Callback Stages and Example}

MATPOWER defines five stages in the execution of a simulation where custom code can be inserted to alter the behavior or data before proceeding to the next stage. This custom code is defined as a set of "callback" functions that are registered via add_userfcn for MATPOWER to call automatically at one of the five stages. Each stage has a name and, by convention, the name of a user-defined callback function ends with the name of the corresponding stage. For example, a callback for the formulation stage that modifies the OPF problem formulation to add reserve requirements could be registered with the following line of code.
```

mpc = add_userfcn(mpc, 'formulation', @userfcn_reserves_formulation);

```

The sections below will describe each stage and the input and output arguments for the corresponding callback function, which vary depending on the stage. An example that employs additional variables, constraints and costs will be used for illustration.

Consider the problem of jointly optimizing the allocation of both energy and reserves, where the reserve requirements are defined as a set of \(n_{r z}\) fixed zonal MW quantities. Let \(Z_{k}\) be the set of generators in zone \(k\) and \(R_{k}\) be the MW reserve requirement for zone \(k\). A new set of variables \(r\) are introduced representing the reserves provided by each generator. The value \(r_{i}\), for generator \(i\), must be nonnegative and is limited above by a user-provided upper bound \(r_{i}^{\max }\) (e.g. a reserve offer quantity) as well as the physical ramp rate \(\Delta_{i}\).
\[
\begin{equation*}
0 \leq r_{i} \leq \min \left(r_{i}^{\max }, \Delta_{i}\right), \quad i=1 \ldots n_{g} \tag{7.2}
\end{equation*}
\]

If the vector \(c\) contains the marginal cost of reserves for each generator, the user defined cost term from (6.34) is simply
\[
\begin{equation*}
f_{u}(\hat{x})=c^{\top} r . \tag{7.3}
\end{equation*}
\]

There are two additional sets of constraints needed. The first ensures that, for each generator, the total amount of energy plus reserve provided does not exceed the capacity of the unit.
\[
\begin{equation*}
p_{g}^{i}+r_{i} \leq p_{g}^{i, \max }, \quad i=1 \ldots n_{g} \tag{7.4}
\end{equation*}
\]

The second requires that the sum of the reserve allocated within each zone \(k\) meets the stated requirements.
\[
\begin{equation*}
\sum_{i \in Z_{k}} r_{i} \geq R_{k}, \quad k=1 \ldots n_{r z} \tag{7.5}
\end{equation*}
\]

Table 7-2: Names Used by Implementation of OPF with Reserves
\begin{tabular}{|c|c|}
\hline name & description \\
\hline mpc & Matpower case struct \\
\hline reserves & additional field in mpc containing input parameters for zonal reserves in the following sub-fields: \\
\hline cost & \(n_{g} \times 1\) vector of reserve costs, \(c\) from (7.3) \\
\hline qty & \(n_{g} \times 1\) vector of reserve quantity upper bounds, \(i^{\text {th }}\) element is \(r_{i}^{\max }\) \\
\hline zones & \(n_{r z} \times n_{g}\) matrix of reserve zone definitions
\[
\text { zones }(\mathrm{k}, \mathrm{j})= \begin{cases}1 & \text { if gen } j \text { belongs to reserve zone } k \quad\left(j \in Z_{k}\right) \\ 0 & \text { otherwise }\left(j \notin Z_{k}\right)\end{cases}
\] \\
\hline req & \(n_{r z} \times 1\) vector of zonal reserve requirements, \(k^{\text {th }}\) element is \(R_{k}\) from (7.5) \\
\hline om & OPF model object, already includes standard OPF setup \\
\hline results & OPF results struct, superset of mpc with additional fields for output data \\
\hline ng & \(n_{g}\), number of generators \\
\hline R & name for new reserve variable block, \(i^{\text {th }}\) element is \(r_{i}\) \\
\hline Pg_plus_R & name for new capacity limit constraint set (7.4) \\
\hline Rreq & name for new reserve requirement constraint set (7.5) \\
\hline
\end{tabular}

Table 7-2 describes some of the variables and names that are used in the example callback function listings in the sections below.

\subsection*{7.3.1 ext2int Callback}

Before doing any simulation of a case, MATPOWER performs some data conversion on the case struct in order to achieve a consistent internal structure, by calling the following.
```

mpc = ext2int(mpc, mpopt);

```

All isolated buses, out-of-service generators and branches are removed, along with any generators or branches connected to isolated buses. The buses are renumbered consecutively, beginning at 1 , and the in-service generators are sorted by increasing bus number. All of the related indexing information and the original data matrices are stored in an order field in the case struct to be used later by int2ext to perform the reverse conversions when the simulation is complete.

The first stage callback is invoked from within the ext2int function immediately after the case data has been converted. Inputs are a MATPOWER case struct (mpc) freshly converted to internal indexing, a MATPOWER options struct mpopt, \({ }^{36}\) and

\footnotetext{
\({ }^{36}\) The mpopt may be empty if ext2int() is called manually without providing it.
}
any (optional) args value supplied when the callback was registered via add_userfcn. Output is the (presumably updated) mpc. This is typically used to reorder any input arguments that may be needed in internal ordering by the formulation stage. The example shows how e2i_field can also be used, with a case struct that has already been converted to internal indexing, to convert other data structures by passing in 2 or 3 extra parameters in addition to the case struct. In this case, it automatically converts the input data in the qty, cost and zones fields of mpc.reserves to be consistent with the internal generator ordering, where off-line generators have been eliminated and the on-line generators are sorted in order of increasing bus number. Notice that it is the second dimension (columns) of mpc.reserves.zones that is being re-ordered. See the on-line help for e2i_field and e2i_data for more details on what all they can do.
```

function mpc = userfcn_reserves_ext2int(mpc, mpopt, args)
mpc = e2i_field(mpc, {'reserves', 'qty'}, 'gen');
mpc = e2i_field(mpc, {'reserves', 'cost'}, 'gen');
mpc = e2i_field(mpc, {'reserves', 'zones'}, 'gen', 2);

```

This stage is also a good place to check the consistency of any additional input data required by the extension and throw an error if something is missing or not as expected.

\subsection*{7.3.2 formulation Callback}

This stage is called at the end of opf setup after the OPF Model (om) object has been initialized with the standard OPF formulation, but before calling the solver. This is the ideal place for modifying the problem formulation with additional variables, constraints and costs, using the add_var, add_lin_constraint, add_nln_constraint, add_quad_cost, add_nln_cost and add_legacy_cost methods of the OPF Model object. \({ }^{37}\) Inputs are the om object, the MATPOWER options struct mpopt and any (optional) args supplied when the callback was registered via add_userfen. Output is the updated om object.

The om object contains both the original Matpower case data as well as all of the indexing data for the variables and constraints of the standard OPF formulation.

\footnotetext{
\({ }^{37}\) It is perfectly legitimate to register more than one callback per stage, such as when enabling multiple independent OPF extensions. In this case, the callbacks are executed in the order they were registered with add_userfcn. E.g. when the second and subsequent formulation callbacks are invoked, the om object will reflect any modifications performed by earlier formulation callbacks.
}

See the on-line help for opf_model and opt_model for more details on the OPF model object and the methods available for manipulating and accessing it.

In the example code, a new variable block named R with \(n_{g}\) elements and the limits from (7.2) is added to the model via the add_var method. Similarly, two linear constraint blocks named Pg_plus_R and Rreq, implementing (7.4) and (7.5), respectively, are added via the add_lin_constraint method. And finally, the add_quad_cost method is used to add to the model a quadratic (actually linear) cost block corresponding to (7.3).

Notice that the last argument to add_lin_constraint and add_quad_cost allows the constraints and costs to be defined only in terms of the relevant parts of the optimization variable \(\hat{x}\). For example, the A matrix for the \(\operatorname{Pg}-\mathrm{plus} \mathrm{R}\) constraint contains only columns corresponding to real power generation (Pg) and reserves ( R ) and need not bother with voltages, reactive power injections, etc. As illustrated in Figure 7-1, this allows the same code to be used with both the AC OPF, where \(\hat{x}\) includes \(V_{m}\) and \(Q_{g}\), and the DC OPF where it does not. This code is also independent of any additional variables that may have been added by Matpower (e.g. \(y\) variables from Matpower's CCV handling of piece-wise linear costs) or by the user via previous formulation callbacks. MATPOWER will place the constraint and cost matrix blocks in the appropriate place when it constructs the aggregated constraint and cost matrices at run-time. This is an important feature that enables independently developed MatPOWER OPF extensions to work together.
```

function om = userfcn_reserves_formulation(om, mpopt, args)
%% initialize some things
define_constants;
mpc = om.get_mpc();
r = mpc.reserves;
ng = size(mpc.gen, 1); %% number of on-line gens
%% variable bounds
Rmin = zeros(ng, 1); %% bound below by 0
Rmax = r.qty; %% bound above by stated max reserve qty ...
k = find(mpc.gen(:, RAMP_10) > 0 \& mpc.gen(:, RAMP_10) < Rmax);
Rmax (k) = mpc.gen(k, RAMP_10); %% ... and ramp rate
Rmax = Rmax / mpc.baseMVA;
%% constraints
I = speye(ng); %% identity matrix
Ar = [I I];
Pmax = mpc.gen(:, PMAX) / mpc.baseMVA;
lreq = r.req / mpc.baseMVA;
%% cost
Cw = r.cost * mpc.baseMVA; %% per unit cost coefficients
%% add them to the model
om.add_var('R', ng, [], Rmin, Rmax);
om.add_lin_constraint('Pg_plus_R', Ar, [], Pmax, {'Pg', 'R'});
om.add_lin_constraint('Rreq', r.zones, lreq, [], {'R'});
om.add_quad_cost('Rcost', [], Cw, 0, {'R'});

```


Figure 7-1: Adding Constraints Across Subsets of Variables

\subsection*{7.3.3 int2ext Callback}

After the simulation is complete and before the results are printed or saved, MatPOWER converts the case data in the results struct back to external indexing by calling the following.
```

results = int2ext(results, mpopt);

```

This conversion essentially undoes everything that was done by ext2int. Generators are restored to their original ordering, buses to their original numbering and all out-of-service or isolated generators, branches and buses are restored.

This callback is invoked from int2ext immediately before the resulting case is converted from internal back to external indexing. At this point, the simulation has been completed and the results struct, a superset of the original Matpower case struct passed to the OPF, contains all of the results. This results struct is passed to the callback, along with the Matpower options struct mpopt, \({ }^{38}\) and any (optional) args supplied when the callback was registered via add_userfen. The output of the callback is the updated results struct. This is typically used to convert any results to external indexing and populate any corresponding fields in the results struct.

The results struct contains, in addition to the standard OPF results, solution information related to all of the user-defined variables, constraints and costs. Table 7-3 summarizes where the various data is found. Each of the fields listed in the table is actually a struct whose fields correspond to the named sets created by add_var, add_lin_constraint, add_nln_constraint, add_quad_cost, add_nln_cost and add_legacy_cost.

In the example code below, the callback function begins by converting the reserves input data in the resulting case (qty, cost and zones fields of results.reserves) back to external indexing via calls to i2e_field. See the help for i2e_field and i2e_data for more details on how they can be used.

\footnotetext{
\({ }^{38}\) The mpopt may be empty if int2ext() is called manually without providing it.
}

Table 7-3: Results for User-Defined Variables, Constraints and Costs
\begin{tabular}{ll}
\hline name & description \\
\hline results.var.val & final value of user-defined variables \\
results.var.mu.l & shadow price on lower limit of user-defined variables \\
results.var.mu.u & shadow price on upper limit of user-defined variables \\
results.lin.mu.l & shadow price on lower (left-hand) limit of linear constraints \\
results.lin.mu.u & shadow price on upper (right-hand) limit of linear constraints \\
results.nle.lambda & shadow price on nonlinear equality constraints \\
results.nli.mu & shadow price on nonlinear inequality constraints \\
results.cost & final value of legacy user costs \\
results.nlc & final value of general nonlinear costs \\
results.qdc & final value of quadratic costs \\
\hline
\end{tabular}

Then the reserves results of interest are extracted from the appropriate sub-fields of results.var, results.lin and results.cost, converted from per unit to per MW where necessary, and stored with external indexing for the end user in the chosen fields of the results struct.
```

function results = userfcn_reserves_int2ext(results, mpopt, args)
%%----- convert stuff back to external indexing -----
%% convert all reserve parameters (zones, costs, qty, rgens)
results = i2e_field(results, {'reserves', 'qty'}, 'gen');
results = i2e_field(results, {'reserves', 'cost'}, 'gen');
results = i2e_field(results, {'reserves', 'zones'}, 'gen', 2);
r = results.reserves;
ng = size(results.gen, 1); %% number of on-line gens (internal)
ng0 = size(results.order.ext.gen, 1); %% number of gens (external)
%%----- results post-processing -----
%% get the results (per gen reserves, multipliers) with internal gen indexing
%% and convert from p.u. to per MW units
[RO, Rl, Ru] = results.om.params_var('R');
R = results.var.val.R * results.baseMVA;
Rmin = Rl * results.baseMVA;
Rmax = Ru * results.baseMVA;
mu_l = results.var.mu.l.R / results.baseMVA;
mu_u = results.var.mu.u.R / results.baseMVA;
mu_Pmax = results.lin.mu.u.Pg_plus_R / results.baseMVA;
%% store in results in results struct
z = zeros(ng0, 1);
results.reserves.R = i2e_data(results, R, z, 'gen');
results.reserves.Rmin = i2e_data(results, Rmin, z, 'gen');
results.reserves.Rmax = i2e_data(results, Rmax, z, 'gen');
results.reserves.mu.l = i2e_data(results, mu_l, z, 'gen');
results.reserves.mu.u = i2e_data(results, mu_u, z, 'gen');
results.reserves.mu.Pmax = i2e_data(results, mu_Pmax, z, 'gen');
results.reserves.prc = z;
for k = 1:ng0
iz = find(r.zones(:, k));
results.reserves.prc(k) = sum(results.lin.mu.l.Rreq(iz)) / results.baseMVA;
end
results.reserves.totalcost = results.cost.Rcost;

```

\subsection*{7.3.4 printpf Callback}

The pretty-printing of the standard OPF output is done via a call to printpf after the case has been converted back to external indexing. This callback is invoked from within printpf after the pretty-printing of the standard OPF output. Inputs are the results struct, the file descriptor to write to, a MATPOWER options struct, and any (optional) args supplied via add_userfcn. Output is the results struct. This is typically used for any additional pretty-printing of results.

In this example, the out.all flag in the options struct is checked before printing anything. If it is non-zero, the reserve quantities and prices for each unit are printed first, followed by the per-zone summaries. An additional table with reserve limit shadow prices might also be included.
```

function results = userfcn_reserves_printpf(results, fd, mpopt, args)
%% define named indices into data matrices
[GEN_BUS, PG, QG, QMAX, QMIN, VG, MBASE, GEN_STATUS, PMAX, PMIN, ...
MU_PMAX, MU_PMIN, MU_QMAX, MU_QMIN, PC1, PC2, QC1MIN, QC1MAX, ...
QC2MIN, QC2MAX, RAMP_AGC, RAMP_10, RAMP_30, RAMP_Q, APF] = idx_gen;
%%----- print results -----
r = results.reserves;
ng = length(r.R);
nrz = size(r.req, 1);
if mpopt.out.all ~}=
fprintf(fd, '\n===========================================================');
fprintf(fd, '\n| Reserves |');
fprintf(fd, '\n==========================================================');
fprintf(fd, '\n Gen Bus Status Reserves Price');
fprintf(fd, '\n \# \# (MW) (\$/MW)');
fprintf(fd, '\n---- ----- ------ -------- ------------);
for k = 1:ng
fprintf(fd, '\n%3d %6d %2d ', k, results.gen(k, GEN_BUS), ...
results.gen(k, GEN_STATUS));
if results.gen(k, GEN_STATUS) > 0 \&\& abs(results.reserves.R(k)) > 1e-6
fprintf(fd, '%10.2f', results.reserves.R(k));
else
fprintf(fd, ' - ');
end
fprintf(fd, '%10.2f ', results.reserves.prc(k));
end
fprintf(fd, '\n --------');
fprintf(fd, '\n Total:%10.2f Total Cost: $%.2f', ...
        sum(results.reserves.R(r.igr)), results.reserves.totalcost);
    fprintf(fd, '\n');
    fprintf(fd, '\nZone Reserves Price ');
    fprintf(fd, '\n # (MW) ($/MW) ');
fprintf(fd, '\n---- --------- --------');
for k = 1:nrz
iz = find(r.zones(k, :)); %% gens in zone k
fprintf(fd, '\n%3d%10.2f%10.2f', k, sum(results.reserves.R(iz)), ...
results.lin.mu.l.Rreq(k) / results.baseMVA);
end
fprintf(fd, '\n');
%% print binding reserve limit multipliers ...
end

```

\subsection*{7.3.5 savecase Callback}

The savecase is used to save a Matpower case struct to an M-file, for example, to save the results of an OPF run. The savecase callback is invoked from savecase after printing all of the other data to the file. Inputs are the case struct, the file descriptor to write to, the variable prefix (typically 'mpc.') and any (optional) args supplied via add_userfen. Output is the case struct. The purpose of this callback is to write any non-standard case struct fields to the case file.

In this example, the zones, req, cost and qty fields of mpc.reserves are written to the M-file. This ensures that a case with reserve data, if it is loaded via loadcase, possibly run, then saved via savecase, will not lose the data in the reserves field. This callback could also include the saving of the output fields if present. The contributed serialize function \({ }^{39}\) can be very useful for this purpose.

\footnotetext{
\({ }^{39}\) http://www.mathworks.com/matlabcentral/fileexchange/1206
}
```

function mpc = userfcn_reserves_savecase(mpc, fd, prefix, args)
%
% mpc = userfcn_reserves_savecase(mpc, fd, mpopt, args)
%
% This is the 'savecase' stage userfcn callback that prints the M-file
% code to save the 'reserves' field in the case file. It expects a
% MATPOWER case struct (mpc), a file descriptor and variable prefix
% (usually 'mpc.'). The optional args are not currently used.
r = mpc.reserves;
fprintf(fd, '\n%%%%%----- Reserve Data -----%%%%%\n');
fprintf(fd, '%%%% reserve zones, element i, j is 1 iff gen j is in zone i\n');
fprintf(fd, '%sreserves.zones = [\n', prefix);
template = '';
for i = 1:size(r.zones, 2)
template = [template, '\t%d'];
end
template = [template, ';\n'];
fprintf(fd, template, r.zones.');
fprintf(fd, '];\n');
fprintf(fd, '\n%%%% reserve requirements for each zone in MW\n');
fprintf(fd, '%sreserves.req = [\t%%', prefix, r.req(1));
if length(r.req) > 1
fprintf(fd, ';\t%g', r.req(2:end));
end
fprintf(fd, '\t];\n');
fprintf(fd, '\n%%%% reserve costs in \$/MW for each gen\n');
fprintf(fd, '%sreserves.cost = [\t%%', prefix, r.cost(1));
if length(r.cost) > 1
fprintf(fd, ';\t%g', r.cost(2:end));
end
fprintf(fd, '\t];\n');
if isfield(r, 'qty')
fprintf(fd, '\n%%%%% max reserve quantities for each gen\n');
fprintf(fd, '%sreserves.qty = [\t%%', prefix, r.qty(1));
if length(r.qty) > 1
fprintf(fd, ';\t%g', r.qty(2:end));
end
fprintf(fd, '\t];\n');
end
%% save output fields for solved case ...

```

\subsection*{7.4 Registering the Callbacks}

As seen in the fixed zonal reserve example, adding a single extension to the standard OPF formulation is often best accomplished by a set of callback functions. A typical use case might be to run a given case with and without the reserve requirements active, so a simple method for enabling and disabling the whole set of callbacks as a single unit is needed.

The recommended method is to define all of the callbacks in a single file containing a "toggle" function that registers or removes all of the callbacks depending on whether the value of the second argument is 'on' or 'off '. The state of the registration of any callbacks is stored directly in the mpc struct. In our example, the toggle_reserves.m file contains the toggle_reserves function as well as the five callback functions.
```

function mpc = toggle_reserves(mpc, on_off)
%TOGGLE_RESERVES Enable, disable or check status of fixed reserve requirements.
% MPC = TOGGLE_RESERVES(MPC, 'on')
% MPC = TOGGLE_RESERVES(MPC, 'off')
% T_F = TOGGLE_RESERVES(MPC, 'status')
if strcmp(upper(on_off), 'ON')
% <code to check for required 'reserves' fields in mpc>
%% add callback functions
mpc = add_userfcn(mpc, 'ext2int', @userfcn_reserves_ext2int);
mpc = add_userfcn(mpc, 'formulation', Quserfcn_reserves_formulation);
mpc = add_userfcn(mpc, 'int2ext', @userfcn_reserves_int2ext);
mpc = add_userfcn(mpc, 'printpf', @userfcn_reserves_printpf);
mpc = add_userfcn(mpc, 'savecase', @userfcn_reserves_savecase);
mpc.userfcn.status.dcline = 1;
elseif strcmp(upper(on_off), 'OFF')
mpc = remove_userfcn(mpc, 'savecase', @userfcn_reserves_savecase);
mpc = remove_userfcn(mpc, 'printpf', @userfcn_reserves_printpf);
mpc = remove_userfcn(mpc, 'int2ext', @userfcn_reserves_int2ext);
mpc = remove_userfcn(mpc, 'formulation', @userfcn_reserves_formulation);
mpc = remove_userfcn(mpc, 'ext2int', @userfcn_reserves_ext2int);
mpc.userfcn.status.dcline = 0;
elseif strcmp(upper(on_off), 'STATUS')
if isfield(mpc, 'userfcn') \&\& isfield(mpc.userfcn, 'status') \&\& ...
isfield(mpc.userfcn.status, 'dcline')
mpc = mpc.userfcn.status.dcline;
else
mpc = 0;
end
else
error('toggle_dcline: 2nd argument must be ''on'', ''off'' or ''status''');
end

```

Running a case that includes the fixed reserves requirements is as simple as loading the case, turning on reserves and running it.
```

mpc = loadcase('t_case30_userfcns');
mpc = toggle_reserves(mpc, 'on');
results = runopf(mpc);

```

\subsection*{7.5 Summary}

The five callback stages currently defined by Matpower are summarized in Table 7-4.

Table 7-4: Callback Functions
\begin{tabular}{|c|c|c|}
\hline name & invoked... & typical use \\
\hline ext2int & ...from ext2int immediately after case data is converted from external to internal indexing. & Check consistency of input data, convert to internal indexing. \\
\hline formulation & ...from opf after OPF Model (om) object is initialized with standard OPF formulation. & Modify OPF formulation, by adding user-defined variables, constraints, costs. \\
\hline int2ext & ...from int2ext immediately before case data is converted from internal back to external indexing. & Convert data back to external indexing, populate any additional fields in the results struct. \\
\hline printpf & ...from printpf after pretty-printing the standard OPF output. & Pretty-print any results not included in standard OPF. \\
\hline savecase & ...from savecase after printing all of the other case data to the file. & Write non-standard case struct fields to the case file. \\
\hline
\end{tabular}

\subsection*{7.6 Example Extensions}

MatPOWER includes three OPF extensions implementing via callbacks, respectively, the co-optimization of energy and reserves, interface flow limits and dispatchable DC transmission lines.

\subsection*{7.6.1 Fixed Zonal Reserves}

This extension is a more complete version of the example of fixed zonal reserve requirements used for illustration above in Sections 7.3 and 7.4. The details of the extensions to the standard OPF problem are given in equations (7.2)-(7.5) and a description of the relevant input and output data structures is summarized in Tables 7-5 and 7-6, respectively.

The code for implementing the callbacks can be found in toggle_reserves. A wrapper around runopf that turns on this extension before running the OPF is

Table 7-5: Input Data Structures for Fixed Zonal Reserves
\begin{tabular}{|c|c|}
\hline name & description \\
\hline mpc & Matpower case struct \\
\hline reserves & additional field in mpc containing input parameters for zonal reserves in the following sub-fields: \\
\hline cost & \(\left(n_{g}\right.\) or \(\left.n_{g r}{ }^{\dagger}\right) \times 1\) vector of reserve costs in \$/MW, c from (7.3) \\
\hline qty & ( \(n_{g}\) or \(n_{g r}{ }^{\dagger}\) ) \(\times 1\) vector of reserve quantity upper bounds in MW, \(i^{\text {th }}\) element is \(r_{i}^{\max }\) \\
\hline zones & \[
\begin{aligned}
& n_{r z} \times n_{g} \text { matrix of reserve zone definitions } \\
& \text { zones }(\mathrm{k}, \mathrm{j})= \begin{cases}1 & \text { if gen } j \text { belongs to reserve zone } k \quad\left(j \in Z_{k}\right) \\
0 & \text { otherwise }\left(j \notin Z_{k}\right)\end{cases}
\end{aligned}
\] \\
\hline req & \(n_{r z} \times 1\) vector of zonal reserve requirements in MW, \(k^{\text {th }}\) element is \(R_{k}\) from (7.5) \\
\hline
\end{tabular}
\(\dagger\) Here \(n_{g r}\) is the number of generators belonging to at least one reserve zone.
Table 7-6: Output Data Structures for Fixed Zonal Reserves
\begin{tabular}{|c|c|}
\hline name & description \\
\hline results & OPF results struct, superset of mpc with additional fields for output data \\
\hline reserves & zonal reserve data, with results in the following sub-fields:* reserve allocation \(r_{i}\), in MW \\
\hline Rmin & lower bound of \(r_{i}\) in MW from (7.2), i.e. all zeros \\
\hline Rmax & upper bound of \(r_{i}\) in MW from (7.2), i.e. \(\min \left(r_{i}^{\max }, \Delta_{i}\right)\) \\
\hline mu & shadow prices on constraints in \$/MW \\
\hline 1 & shadow price on lower bound of \(r_{i}\) in (7.2) \\
\hline u & shadow price on upper bound of \(r_{i}\) in (7.2) \\
\hline Pmax & shadow price on capacity constraint (7.4) \\
\hline prc & reserve price in \(\$ / \mathrm{MW}\), computed for unit \(i\) as the sum of the shadow prices of constraints \(k\) in (7.5) in which unit \(i\) participates \(\left(k \mid i \in Z_{k}\right)\) \\
\hline
\end{tabular}
\(\ddagger\) All zonal reserve result sub-fields are \(n_{g} \times 1\) vectors, where the values are set to zero for units not participating in the provision of reserves.
provided in runopf_w_res, allowing you to run a case with an appropriate reserves field, such as t_case30_userfcns, as follows.
```

results = runopf_w_res('t_case30_userfcns');

```

See help runopf_w_res and help toggle_reserves for more information. Examples of using this extension and a case file defining the necessary input data can be found in t_opf_userfens and t_case30_userfcns, respectively. Additional tests for runopf_w_res are included in t_runopf_w_res.

\subsection*{7.6.2 Interface Flow Limits}

This extension adds interface flow limits based on flows computed from a DC network model. It is implemented in toggle_iflims. A flow interface \(k\) is defined as a set \(\mathcal{B}_{k}\) of branch indices \(i\) and a direction for each branch. If \(p_{i}\) represents the real power flow ("from" bus \(\rightarrow\) "to" bus) in branch \(i\) and \(d_{i}\) is equal to 1 or -1 to indicate the direction, \({ }^{40}\) then the interface flow \(f_{k}\) for interface \(k\) is defined as
\[
\begin{equation*}
f_{k}(\Theta)=\sum_{i \in \mathcal{B}_{k}} d_{i} p_{i}(\Theta), \tag{7.6}
\end{equation*}
\]
where each branch flow \(p_{i}\) is an approximation calculated as a linear function of the bus voltage angles based on the DC power flow model from equation (3.29).

This extension adds to the OPF problem a set of \(n_{\text {if }}\) doubly-bounded constraints on these flows.
\[
\begin{equation*}
F_{k}^{\min } \leq f_{k}(\Theta) \leq F_{k}^{\max } \quad \forall k \in \mathcal{I}_{f} \tag{7.7}
\end{equation*}
\]
where \(F_{k}^{\min }\) and \(F_{k}^{\max }\) are the specified lower and upper bounds on the interface flow, and \(\mathcal{I}_{f}\) is a the set indices of interfaces whose flow limits are to be enforced.

The data for the problem is specified in an additional if field in the Matpower case struct mpc. This field is itself a struct with two sub-fields, map and lims, used for input data, and two others, P and mu, used for output data. The format of this data is described in detail in Tables 7-7 and 7-8.

Table 7-7: Input Data Structures for Interface Flow Limits
\begin{tabular}{|c|c|}
\hline name & description \\
\hline mpc & MATPOWER case struct \\
\hline if & additional field in mpc containing input parameters for interface flow limits in the following sub-fields: \\
\hline map & \(\left(\sum_{k} n_{k}\right) \times 2\) matrix defining the interfaces, where \(n_{k}\) is the number branches that belong to interface \(k\). The \(n_{k}\) branches of interface \(k\) are defined by \(n_{k}\) rows in the matrix, where the first column in each is equal to \(k\) and the second is equal to the corresponding branch index \(i\) multiplied by \(d_{i}\) to indicate the direction. \\
\hline lims & \(n_{\text {if }} \times 3\) matrix of interface limits, where \(n_{\text {if }}\) is the number of interface limits to be enforced. The first column is the index \(k\) of the interface, and the second and third columns are \(F_{k}^{\min }\) and \(F_{k}^{\max }\), the lower and upper limits respectively, on the DC model flow limits (in MW) for the interface. \\
\hline
\end{tabular}

\footnotetext{
\({ }^{40}\) If \(d_{i}=1\), the definitions of the positive flow direction for the branch and the interface are the same. If \(d_{i}=-1\), they are opposite.
}

Table 7-8: Output Data Structures for Interface Flow Limits
\begin{tabular}{cl}
\hline name & description \\
\hline results & OPF results struct, superset of mpc with additional fields for output data \\
if & \begin{tabular}{l} 
additional field in results containing output parameters for interface flow \\
\\
P
\end{tabular} \\
& \(n_{\text {if }} \times 1\) vector of actual flow in MW across the corresponding interface (as \\
mu.l & measured at the "from" end of associated branches) \\
mu.u & \(n_{\text {if }} \times 1\) vector of shadow prices on lower flow limits \((u / \mathrm{MW})^{\dagger}\) \\
\(n_{\text {if }} \times 1\) vector of shadow prices on upper flow limits \((u / \mathrm{MW})^{\dagger}\) \\
\hline
\end{tabular}
\(\dagger\) Here we assume the objective function has units \(u\).

See help toggle_iflims for more information. Examples of using this extension and a case file defining the necessary input data for it can be found in t_opf_userfens and t_case30_userfcns, respectively. Note that, while this extension can be used for AC OPF problems, the actual AC interface flows will not necessarily be limited to the specified values, since it is a DC flow approximation that is used for the constraint.

Running a case that includes the interface flow limits is as simple as loading the case, turning on the extension and running it. Unlike with the reserves extension, MATPOWER does not currently have a wrapper function to automate this.
```

mpc = loadcase('t_case30_userfcns');
mpc = toggle_iflims(mpc, 'on');
results = runopf(mpc);

```

\subsection*{7.6.3 DC Transmission Lines}

Beginning with version 4.1, MATPOWER also includes a simple model for dispatchable DC transmission lines. While the implementation is based on the extensible OPF architecture described above, it can be used for simple power flow problems as well, in which the case the (OPF only) formulation callback is skipped.

A DC line in MATPOWER is modeled as two linked "dummy" generators, as shown in Figures 7-2 and 7-3, one with negative capacity extracting real power from the network at the "from" end of the line and another with positive capacity injecting power into the network at the "to" end. These dummy generators are added by the ext2int callback and removed by the int2ext callback. The real power flow \(p_{f}\) on the DC line at the "from" end is defined to be equal to the negative of the injection of corresponding dummy generator. The flow at the "to" end \(p_{t}\) is defined to be equal to the injection of the corresponding generator.


Figure 7-2: DC Line Model


Figure 7-3: Equivalent "Dummy" Generators

MATPOWER links the values of \(p_{f}\) and \(p_{t}\) using the following relationship, which includes a linear approximation of the real power loss in the line.
\[
\begin{align*}
p_{t} & =p_{f}-p_{\text {loss }} \\
& =p_{f}-\left(l_{0}+l_{1} p_{f}\right) \\
& =\left(1-l_{1}\right) p_{f}-l_{0} \tag{7.8}
\end{align*}
\]

Here the linear coefficient \(l_{1}\) is assumed to be a small ( \(\ll 1\) ) positive number. Obviously, this is not applicable for bi-directional lines, where the flow could go either direction, resulting in decreasing losses for increasing flow in the "to" \(\rightarrow\) "from" direction. There are currently two options for handling bi-directional lines. The first is to use a constant loss model by setting \(l_{1}=0\). The second option is to create two separate but identical lines oriented in opposite directions. In this case, it is important that the lower limit on the flow and the constant term of the loss model \(l_{0}\) be set to zero to ensure that only one of the two lines has non-zero flow at a time. \({ }^{41}\)

Upper and lower bounds on the value of the flow can be specified for each DC line, along with an optional operating cost. It is also assumed that the terminals of the line have a range of reactive power capability that can be used to maintain a voltage setpoint. Just as with a normal generator, the voltage setpoint is only used for simple power flow; the OPF dispatches the voltage anywhere between the lower and upper bounds specified for the bus. Similarly, in a simple power flow the input value for \(p_{f}\) and the corresponding value for \(p_{t}\), computed from (7.8), are used to specify the flow in the line.

Most of the data for DC lines is stored in a dcline field in the Matpower case struct mpc. This field is a matrix similar to the branch matrix, where each row corresponds to a particular DC line. The columns of the matrix are defined in Table B-5 and include connection bus indices, line status, flows, terminal reactive injections, voltage setpoints, limits on power flow and VAr injections, and loss parameters. Also, similar to the branch or gen matrices, some of the columns are used for input values, some for results, and some, such as PF can be either input or output, depending on whether the problem is a simple power flow or an optimal power flow. The idx_dcline function defines a set of constants for use as named column indices for the dcline matrix.

An optional dclinecost matrix, in the same form as gencost, can be used to specify a cost to be applied to \(p_{f}\) in the OPF. If the dclinecost field is not present, the cost is assumed to be zero.

\footnotetext{
\({ }^{41} \mathrm{~A}\) future version may make the handling of this second option automatic.
}

MATPOWER's DC line handling is implemented in toggle_dcline and examples of using it can be found in t_dcline. The case file t_case9_dcline includes some example DC line data. See help toggle_dcline for more information.

Running a case that includes DC lines is as simple as loading the case, turning on the extension and running it. Unlike with the reserves extension, Matpower does not currently have a wrapper function to automate this.
```

mpc = loadcase('t_case9_dcline');
mpc = toggle_dcline(mpc, 'on');
results = runopf(mpc);

```

\subsection*{7.6.4 OPF Soft Limits}

Matpower includes an extension that replaces limits (such as branch flow limits, voltage magnitude bounds, generator limits, etc.) in an optimal power flow with soft limits, that is, limits that can be violated with some linear penalty cost. This can be useful in identifying the cause of infeasibility in some optimal power flow problems. The extension is implemented in toggle_softlims. A limited version for branch flow constraints in a DC model only, was introduced in Matpower 5.0, with a generalization to all of the standard OPF limits added in Matpower 7.0. The soft AC branch flow limit implemention provides an example of user-defined nonlinear constraints.

In general, replacing a hard constraint of the form
\[
\begin{equation*}
h^{i}(x) \leq 0 \tag{7.9}
\end{equation*}
\]
with a soft limit involves introducing a new non-negative variable \(s_{i}\) to represent the violation of the original constraint. The original hard constraint (7.9) is replaced by one in which this violation variable is used to "relax" the constraint by adding it to the right-hand.
\[
\begin{align*}
s^{i} & \geq 0  \tag{7.10}\\
h^{i}(x) & \leq s^{i} . \tag{7.11}
\end{align*}
\]

Finally, a penalty cost on violations is implemented via a linear cost coefficient, \(c_{\mathrm{v}}^{i}\), applied to each violation variable \(s^{i}\), so that the additional user defined cost term from (6.47) for the set of soft limits looks like
\[
\begin{equation*}
f_{u}(\hat{x})=c_{\mathrm{v}}^{\top} s \tag{7.12}
\end{equation*}
\]

Take, as an example, branch flow limits. The flow constraints in (6.9) and (6.10) are rewritten by replacing the right hand side with a flow violation variable \(s_{\text {rate_a }}^{i}\)
\[
\left.\begin{array}{rlrl}
s_{\text {rate_a }}^{i} & \geq 0 & & \forall i \\
h_{f}^{i}\left(\Theta, V_{m}\right) & =\left|f_{f}^{i}\left(\Theta, V_{m}\right)\right|-f^{i, \text { max }} & \leq s_{\text {rate_a }}^{i} & \\
h_{t}^{i}\left(\Theta, V_{m}\right) & =\left|f_{t}^{i}\left(\Theta, V_{m}\right)\right|-f^{i, \max } & \leq s_{\text {rate_a }}^{i} & \tag{7.15}
\end{array}\right\rangle i .
\]

In the case of AC flow constraints, the flow functions \(f_{f}^{i}\) and \(f_{t}^{i}\) are nonlinear and can be either apparent power, active power, or current as shown in (6.11). In the case of DC flow constraints, where there are no losses, these flows can be implemented in terms of the flow at the from end of the line and its negative, both linear functions of the bus voltage angles, based on (6.30) and (6.31).
\[
\begin{align*}
s_{\text {rate_a }}^{i} & \geq 0 & & \forall i  \tag{7.16}\\
B_{f}^{i} \Theta+p_{f, \text { shift }}^{i}-p_{f}^{i, \text { max }} & \leq s_{\text {rate_a }}^{i} & & \forall i  \tag{7.17}\\
-B_{f}^{i} \Theta-p_{f, \text { shift }}^{i}-p_{f}^{i, \text { max }} & \leq s_{\text {rate_a }}^{i} & & \forall i . \tag{7.18}
\end{align*}
\]

The feasible area for the DC case is illustrated in Figure 7-4.


Figure 7-4: Feasible Region for Branch Flow Violation Constraints

MatPOWER implements soft limits for each of the types of constraints listed in Table 7-9, where the formulation for each constraint type is shown.

The parameters for the problem are specified in an additional softlims field in the Matpower case struct mpc. This field is itself a struct where each sub-field

Table 7-9: Soft Limit Formulation
\begin{tabular}{|c|c|c|}
\hline name & hard-limit formulation & soft-limit formulation \\
\hline VMIN, VMAX \({ }^{\dagger}\) & \(v_{m}^{i} \geq v_{m}^{i, \text { min }}, \quad v_{m}^{i} \leq v_{m}^{i, \text { max }}\) & \({ }_{\text {min }} \geq v_{m}^{i, \min }, \quad v_{m}^{i}-s_{\mathrm{vmax}}^{i} \leq v_{m}^{i, \text { max }}\) \\
\hline \multirow[t]{2}{*}{RATE_A \({ }^{\ddagger}\)} & \(\left|f_{f}^{i}\left(\Theta, V_{m}\right)\right| \leq f_{\text {max }}^{i}\) & \(\left|f_{f}^{i}\left(\Theta, V_{m}\right)\right|-s_{\text {rate_a }} \leq f_{\text {max }}^{i}\) \\
\hline & \(\left|f_{t}^{i}\left(\Theta, V_{m}\right)\right| \leq f_{\max }^{i}\) & \(\left|f_{t}^{i}\left(\Theta, V_{m}\right)\right|-s_{\text {rate_a }} \leq f_{\text {max }}^{i}\) \\
\hline PMIN, PMAX & \(p_{g}^{i} \geq p_{g}^{i, \text { min }}, \quad p_{g}^{i} \leq p_{g}^{i, \max }\) & \(p_{g}^{i}+s_{\text {pmin }} \geq p_{g}^{i, \text { min }}, \quad p_{g}^{i}-s_{\text {pmax }} \leq p_{g}^{i, \text { max }}\) \\
\hline QMIN, QMAX \({ }^{\dagger}\) & \(q_{g}^{i} \geq q_{g}^{i, \min }, \quad q_{g}^{i} \leq q_{g}^{i, \max }\) & \(q_{g}^{i}+s_{\mathrm{qmin}} \geq q_{g}^{i, \text { min }}, \quad q_{g}^{i}-s_{\mathrm{qmax}} \leq q_{g}^{i, \max }\) \\
\hline ANGMIN & \(\theta_{f}^{i}-\theta_{t}^{i} \geq \Delta \theta^{i, \text { min }}\) & \(\theta_{f}^{i}-\theta_{t}^{i}+s_{\text {angmin }} \geq \Delta \theta^{i, \text { min }}\) \\
\hline ANGMAX & \(\theta_{f}^{i}-\theta_{t}^{i} \leq \Delta \theta^{i, \max }\) & \(\theta_{f}^{i}-\theta_{t}^{i}-s_{\text {angmax }} \leq \Delta \theta^{i, \text { max }}\) \\
\hline
\end{tabular}
\(\dagger\) Only applicable for the AC OPF.
\(\ddagger\) Example of a user-defined nonlinear constraint in the AC version.
corresponds to the name of one of the limits listed in Table 7-9. For each of the limits, the input parameters are specified in the fields detailed in Table 7-10, with defaults summarized in Table 7-11.

Depending on whether the original constraint in (7.10) is an upper bound or a lower bound, it can be written as one of the following
\[
h^{i}(x)= \begin{cases}\bar{h}^{i}(x)-h^{i, \lim } & \text { (upper bound) }  \tag{7.19}\\ h^{i, \lim }-\bar{h}^{i}(x) & \text { (lower bound) }\end{cases}
\]
where \(h^{i, \text { lim }}\) is the actual value of the hard limit. When introducing a soft limit, this original hard limit can be removed completely or modified to have a new value. MATPOWER provides a several ways of modifying the original hard limit based on fields hl_mod ("hard limit modification") and hl_val ("hard limit value") in the respective limit structures. This new hard limit is implemented via an upper bound on the corresponding violation variable \(s^{i}\). Table 7-12 summarizes the new hard limits and the corresponding bounds on the violation variable as functions of the values of hl_mod and hl_val.

Unspecified limits in the mpc.softlims input parameters are handled differently, depending on the value of the MATPOWER option 'opf.softlims.default' (see Table C-5).
- If the opf.softlims.default option is 1 (default), each limit not explicitly specified, whether missing in mpc.softlims or present but empty, will be initialized as in Table 7-11.
- If the opf. softlims. default option is 0 , limits that are missing in mpc.softlims are treated as though specified with \(\mathrm{hl} \bmod =\) 'none', that is, they are not

Table 7-10: Input Data Structures for OPF Soft Limits
\begin{tabular}{|c|c|}
\hline name & description \\
\hline mpc & MatPower case struct \\
\hline softlims & additional field in mpc containing OPF soft limit input parameters for the possible limits, each of which is optional.* \\
\hline <LIM> & <LIM> refers to the name of the limit, i.e. one of the following: ANGMIN, ANGMAX, RATE_A, PMIN, PMAX, QMIN, QMAX, VMIN, VMAX. Each of these is a struct with input parameters defining the soft limits for this type of constraint in the following optional fields (see Table 7-11 for defaults): \\
\hline idx & \(n_{\mathrm{sl}} \times 1\) vector of row indices for bus, branch or gen matrix, depending on <LIM>, specifying the elements for which corresponding soft limits are to be applied \({ }^{\dagger}\) \\
\hline busnum \({ }^{\ddagger}\) & \(n_{\mathrm{sl}} \times 1\) vector of external bus numbers specifying the buses to which soft limits are to be applied \({ }^{\dagger}\) \\
\hline \multirow[t]{5}{*}{hl_mod \({ }^{\S}\)} & scalar or \(n_{\mathrm{sl}} \times 1\) vector \(c_{\mathrm{v}}\) of linear cost coefficients for limit violation costs string indicating type of modification to original hard limit: \\
\hline & \begin{tabular}{l}
'none' - do not add soft limit, no change to original hard limit \\
'remove' - add soft limit, relax hard limit by removing it completely
\end{tabular} \\
\hline & 'replace' - add soft limit, relax hard limit by replacing original with value specified in hl_val \\
\hline & 'scale' - add soft limit, relax hard limit by scaling original by value specified in hl_val \\
\hline & 'shift' - add soft limit, relax hard limit by shifting original by value specified in hl_val \\
\hline \multirow[t]{2}{*}{hl_val \({ }^{\S}\)} & scalar or \(n_{\mathrm{sl}} \times 1\) vector value used to modify hard limit according to hl_mod. Ignored for 'none' and 'remove', required for 'replace', and optional, with the following defaults, for 'scale' and 'shift': \\
\hline & 'scale' -2 for positive upper limits or negative lower limits, 0.5 otherwise 'shift' - 0.25 for VMAX and VMIN, 10 otherwise \\
\hline
\end{tabular}

\footnotetext{
* For fields not present, the value of the 'opf.softlims.defaults' option determines whether the corresponding limit is unchanged \((=0)\) or converted \((=1\), default) to a soft limit using default values from Table 7-11.
\(\dagger\) For bus constraints, idx overrides busnum if both are provided.
\(\ddagger\) Only applicable for bus constraints, i.e. <LIM> is VMIN or VMAX.
§ Any new, relaxed hard limits are implemented via bounds on the violation variables. See Table 7-12 for details.
}
converted to soft limits at all. On the other hand, the default soft limits of Table \(7-11\) are still applied to any limit that is present in mpc.softlims but empty. For example, mpc.softlims.RATE_A = struct() would result in the default soft branch limits being applied.

Soft limit outputs, summarized in Table 7-13, include the amount and cost of any overloads, that is, violations of the original hard limits. These can be found as additional fields, overload and ov_cost, under each limit in results.softlims. The Kuhn-Tucker multipliers on the soft limit constraints are also included in the

Table 7-11: Default Soft Limit Values
\begin{tabular}{|c|c|}
\hline name & description \\
\hline VMIN & voltage magnitude lower bound \\
\hline idx & all buses \\
\hline cost & \$100,000/p.u. \({ }^{\dagger}\) \\
\hline hl_mod & 'replace' \\
\hline hl_val & 0 \\
\hline VMAX & voltage magnitude upper bound \\
\hline idx & all buses \\
\hline cost & \$100,000/p.u. \({ }^{\dagger}\) \\
\hline hl_mod & 'remove' \\
\hline RATE_A & branch flow limit \\
\hline idx & all on-line branches with bounded flow limits \\
\hline cost & \$1000/MVA, \$1000/MW, or \$1000/MA \({ }^{\dagger}\) depending on 'opf.flow_lim' \\
\hline hl_mod & 'remove' \\
\hline PMIN & generator active power lower bound \\
\hline idx & all on-line generators, excluding dispatchable loads \\
\hline cost & \$1000/MW \({ }^{\dagger}\) \\
\hline hl_mod & 'replace' \\
\hline hl_val & 0 if \(P_{g}^{i, \min } \geq 0,-\infty\) otherwise \\
\hline PMAX, QMIN, QMAX & generator active power upper bound, reactive power lower/upper bounds \\
\hline idx & all on-line generators, excluding dispatchable loads \\
\hline cost & \(\$ 1000 / \mathrm{MW}\) or \(\$ 1000 / \mathrm{MVAr}^{\dagger}\) \\
\hline hl_mod & 'remove' \\
\hline ANGMIN, ANGMAX & branch angle difference lower/upper bounds \\
\hline idx & all on-line branches with angle difference limits, \(\Delta \theta\), where \(|\Delta \theta|<360^{\circ}\) \\
\hline cost & \$1000/deg \({ }^{\dagger}\) \\
\hline hl mod & 'remove' \\
\hline
\end{tabular}
\(\dagger\) Unless the maximum marginal cost at \(P^{\max }\) across all online generators, which we call \(c_{g}^{\max }\), exceeds \(\$ 1000 / \mathrm{MWh}\), in which case the numerical value of the default cost is \(c_{g}^{\max }\) instead of 1000 (or \(100 \cdot c_{g}^{\max }\) instead of 100,000 , for VMIN or VMAX).

Table 7-12: Possible Hard-Limit Modifications
\begin{tabular}{lcc}
\hline h_mod & new hard limit \(h_{\text {new }}^{i, \text { lim }}\) & upper bound on violation variable \(s^{i}\) \\
\hline 'remove' & \(\pm \infty^{\dagger}\) & \(\infty\) \\
'replace' & hl_val & \(\mid \mathrm{hl}\) _val \(-h_{\text {orig }}^{i, \text { lim }} \mid\) \\
'scale' & hl_val \(\cdot h_{\text {orig }}^{i, \text { lim }}\) & \(\left|\left(\mathrm{hl} \mathrm{\_val}-1\right) \cdot h_{\text {orig }}^{i, \text { lim }}\right|\) \\
'shift' & \(h_{\text {orig }}^{i, \text { lim }} \pm \mathrm{hl} \mathrm{\_val}{ }^{\dagger}\) & hl_val \\
\hline
\end{tabular}
\(\dagger\) Plus sign for upper bounds and minus sign for lower bounds.
usual columns of the corresponding matrix in the results, e.g. bus(:, MU_VMAX), branch(:, MU_SF), etc. When there are no violations, these shadow prices are the same as if hard limits were imposed. When there is a violation, they are equal to the \(\operatorname{cost} c_{\mathrm{v}}\) associated with the violation variable, except in the case where a modified hard limit is also binding, in which case it also reflects the shadow price on this modified hard constraint.

Table 7-13: Output Data Structures for OPF Soft Limits
\begin{tabular}{|c|c|}
\hline name & description \\
\hline results & Matpower case struct \\
\hline softlims & additional field in results containing OPF soft limit outputs for each limit type \\
\hline <LIM> & <LIM> refers to the name of the limit, i.e. one of the following: ANGMIN, ANGMAX, RATE_A, PMIN, PMAX, QMIN, QMAX, VMIN, VMAX. Each of these is a struct with the following output fields: \\
\hline overload \({ }^{\dagger}\) & \(n_{b / l / g} \times 1\) vector of overload quantities for this limit (amount of original hard limit violation) \\
\hline ov_cost \({ }^{\dagger}\) & \(n_{b / l / g} \times 1\) vector of overload penalty costs for this limit \\
\hline branch(:, MU_ANGMIN) & Kuhn-Tucker multipliers on angle difference lower bounds \({ }^{\ddagger}\) \\
\hline branch(:, MU_ANGMAX) & Kuhn-Tucker multipliers on angle difference upper bounds \({ }^{\ddagger}\) \\
\hline branch(:, MU_SF) & Kuhn-Tucker multipliers on branch flow limits (from end) \({ }^{\ddagger}\) \\
\hline branch(:, MU_ST) & Kuhn-Tucker multipliers on branch flow limits (to end) \({ }^{\ddagger}\) \\
\hline gen(:, MU_PMIN) & Kuhn-Tucker multipliers on active gen lower bounds \({ }^{\ddagger}\) \\
\hline gen(:, MU_PMAX) & Kuhn-Tucker multipliers on active gen upper bounds \({ }^{\ddagger}\) \\
\hline gen(:, MU_QMIN) & Kuhn-Tucker multipliers on reactive gen lower bounds \({ }^{\ddagger}\) \\
\hline gen(:, MU_QMAX) & Kuhn-Tucker multipliers on reactive gen upper bounds \({ }^{\ddagger}\) \\
\hline bus(:, MU_VMIN) & Kuhn-Tucker multipliers on voltage magnitude lower bounds \({ }^{\ddagger}\) \\
\hline bus(:, MU_VMAX) & Kuhn-Tucker multipliers on voltage magnitude upper bounds \({ }^{\ddagger}\) \\
\hline
\end{tabular}
\(\dagger\) The dimensions of overload and ov_cost correspond to all buses, branches or generators (depending on <LIM>), not just those whose limits were converted to soft limits. Entries corresponding to those not included (implicitly or explicitly) in idx are set to zero.
\(\dagger\) For limits that have been converted to soft limits, these are the shadow prices on the soft limit constraints. When there is no violation of the soft limit, this shadow price is the same as it would be for the original hard limit. When there is a violation, it is equal to the corresponding user-supplied violation cost \(c_{\mathrm{v}}^{i}\), unless an updated hard constraint is also binding.

See help toggle_softlims for more information on this extension. Examples of using this extension can be found in t_opf_softlims. Running a case that includes the default soft limits is as simple as loading the case, turning on the extension and running it. Unlike with the reserves extension, Matpower does not currently have a wrapper function to automate this.
```

mpc = loadcase('case2383wp');
mpc = toggle_softlims(mpc, 'on');
results = runopf(mpc, mpopt);

```

\section*{8 Unit De-commitment Algorithm}

The standard OPF formulation described in the previous section has no mechanism for completely shutting down generators which are very expensive to operate. Instead they are simply dispatched at their minimum generation limits. Matpower includes the capability to run an optimal power flow combined with a unit de-commitment for a single time period, which allows it to shut down these expensive units and find a least cost commitment and dispatch. To run this for case30, for example, type:
```

>> runuopf('case30')

```

By default, runuopf is based on the AC optimal power flow problem. To run a DC OPF, the model option must be set to 'DC'. For convenience, MATPOWER provides a function runduopf which is simply a wrapper that sets the model option to 'DC' before calling runuopf.

Matpower uses an algorithm similar to dynamic programming to handle the de-commitment. It proceeds through a sequence of stages, where stage \(N\) has \(N\) generators shut down, starting with \(N=0\), as follows:

Step 1: Begin at stage zero \((N=0)\), assuming all generators are on-line with all limits in place.

Step 2: If the sum of the minimum generation limits for all on-line generators is less than the total system demand, then go to Step 3. Otherwise, go to the next stage, \(N=N+1\), shut down the generator whose average per-MW cost of operating at its minimum generation limit is greatest and repeat Step 2.

Step 3: Solve a normal OPF. Save the solution as the current best.
Step 4: Go to the next stage, \(N=N+1\). Using the best solution from the previous stage as the base case for this stage, form a candidate list of generators with minimum generation limits binding. If there are no candidates, skip to Step 6.

Step 5: For each generator on the candidate list, solve an OPF to find the total system cost with this generator shut down. Replace the current best solution with this one if it has a lower cost. If any of the candidate solutions produced an improvement, return to Step 4.

Step 6: Return the current best solution as the final solution.

It should be noted that the method employed here is simply a heuristic. It does not guarantee that the least cost commitment of generators will be found. It is also rather computationally expensive for larger systems and was implemented as a simple way to allow an OPF-based "smart-market", such as described in Appendix F, the option to reject expensive offers while respecting the minimum generation limits on generators.

\section*{9 Miscellaneous Matpower Functions}

This section describes a number of additional Matpower functions that users may find useful. The descriptions here are simply brief summaries, so please use the Matlab help function to get the full details on each function.

\subsection*{9.1 Input/Output Functions}

\subsection*{9.1.1 loadcase}
```

mpc = loadcase(casefile)

```

The loadcase function provides the canonical way of loading a MATPOWER case from a file or struct. It takes as input either a struct or the name of an M-file or MAT-file in the Matlab path (casefile) and returns a standard Matpower case struct (mpc). It can also convert from the older version 1 case file format to the current format. This function allows a case to be loaded, and potentially modified, before calling one of the main simulation functions such as runpf or runopf.

\subsection*{9.1.2 savecase}
```

savecase(fname, mpc)
savecase(fname, mpc, version)
savecase(fname, comment, mpc)
savecase(fname, comment, mpc, version)
fname = savecase(fname, ...)

```

The savecase function writes out a Matpower case file, given a name for the file to be created or overwritten (fname), and a MATPOWER case struct (mpc). If fname ends with '.mat' it saves the case as a MAT-file, otherwise it saves it as an M-file. Optionally returns the filename, with extension added if necessary. The optional comment argument is either string (single line comment) or a cell array of strings which are inserted as comments in the help section of the file. If the optional version argument is ' 1 ' it will modify the data matrices to version 1 format before saving.

\subsection*{9.1.3 cdf2mpc}
```

mpc = cdf2mpc(cdf_file_name)
mpc = cdf2mpc(cdf_file_name, verbose)
mpc = cdf2mpc(cdf_file_name, mpc_name)
mpc = cdf2mpc(cdf_file_name, mpc_name, verbose)
[mpc, warnings] = cdf2mpc(cdf_file_name, ...)

```

The cdf2mpc function converts an IEEE Common Data Format (CDF) data file into a Matpower case struct. Given an optional file name mpc_name, it can save the converted case to a Matpower case file. Warnings generated during the conversion process can be optionally returned in the warnings argument.

Since the IEEE CDF format does not contain all of the data needed to run an optimal power flow, some data, such as voltage limits, generator limits and generator costs are created by cdf2mpc. See help cdf2mpc for details.

\subsection*{9.1.4 psse2mpc}
```

mpc = psse2mpc(rawfile_name)
mpc = psse2mpc(rawfile_name, verbose)
mpc = psse2mpc(rawfile_name, verbose, rev)
mpc = psse2mpc(rawfile_name, mpc_name)
mpc = psse2mpc(rawfile_name, mpc_name, verbose)
mpc = psse2mpc(rawfile_name, mpc_name, verbose, rev)
[mpc, warnings] = psse2mpc(rawfile_name, ...)

```

The psse2mpc function converts a PSS/E RAW data file into a MATPOWER case struct. Given an optional file name mpc_name, it can save the converted case to a Matpower case file. Warnings generated during the conversion process can be optionally returned in the warnings argument. By default, psse2mpc attempts to determine the revision of the PSS/E RAW file from the contents, but the user can specify an explicit revision number to use in the optional rev argument.

\subsection*{9.1.5 save2psse}
```

save2psse(fname, mpc)
fname_out = save2psse(fname, mpc)

```

The save2psse function saves a MATPOWER case struct mpc as a PSS/E RAW file. The fname parameter is a string containing the name of the file to be created or overwritten. If fname does not include a file extension, codeq.raw will be added. Optionally returns the, possibly updated, filename. Currently exports to RAW format Rev 33.

\subsection*{9.2 System Information}

\subsection*{9.2.1 case_info}
```

case_info(mpc)
case_info(mpc, fd)
[groups, isolated] = case_info(mpc)

```

The case_info function prints out detailed information about a Matpower case, including connectivity information, summarizing the generation, load and other data by interconnected island. It can optionally print the output to an open file, whose file identifier (as returned by fopen) is specified in the optional second parameter fd. Optional return arguments include groups and isolated buses, as returned by the find_islands function.

\subsection*{9.2.2 compare_case}
compare_case(mpc1, mpc2)

Compares the bus, branch and gen matrices of two Matpower cases and prints a summary of the differences. For each column of the matrix it prints the maximum of any non-zero differences.

\subsection*{9.2.3 find_islands}
```

groups = find_islands(mpc)
[groups, isolated] = find_islands(mpc)

```

The find_islands function returns the islands in a network. The return value groups is a cell array of vectors of the bus indices for each island. The second and optional return value isolated is a vector of indices of isolated buses that have no connecting branches.

\subsection*{9.2.4 get_losses}
```

loss = get_losses(results)
loss = get_losses(baseMVA, bus, branch)
[loss, chg] = get_losses(results)
[loss, fchg, tchg] = get_losses(results)
[loss, fchg, tchg, dloss_dv] = get_losses(results)
[loss, fchg, tchg, dloss_dv, dchg_dvm] = get_losses(results)

```

The get_losses function computes branch series losses, and optionally reactive injections from line charging, as functions of bus voltages and branch parameters, using the following formulae for a branch, as described in Section 3.2, connecting bus \(f\) to bus \(t\) :
\[
\begin{align*}
\operatorname{loss}_{i} & =\frac{\left|\frac{v_{f}}{\tau e^{j s_{\text {shift }}}}-v_{t}\right|^{2}}{r_{s}-j x_{s}}  \tag{9.1}\\
f_{\text {chg }} & =\left|\frac{v_{f}}{\tau e^{j \theta_{\text {shift }}}}\right|^{2} \frac{b_{c}}{2}  \tag{9.2}\\
t_{\text {chg }} & =\left|v_{t}\right|^{2} \frac{b_{c}}{2} \tag{9.3}
\end{align*}
\]

It can also optionally compute the partial derivatives of the line losses and reactive charging injections with respect to voltage angles and magnitudes.

\subsection*{9.2.5 margcost}
```

marginalcost = margcost(gencost, Pg)

```

The margcost function computes the marginal cost for generators given a matrix in gencost format and a column vector or matrix of generation levels. The return value has the same dimensions as Pg. Each row of gencost is used to evaluate the cost at the output levels specified in the corresponding row of Pg. The rows of gencost can specify either polynomial or piecewise linear costs and need not be uniform.

\subsection*{9.2.6 isload}
```

TorF = isload(gen)

```

The isload function returns a column vector of 1's and 0's. The 1's correspond to rows of the gen matrix which represent dispatchable loads. The current test is \(P_{\text {min }}<0\) and \(P_{\text {max }}=0\).

\subsection*{9.2.7 loadshed}
```

shed = loadshed(gen)
shed = loadshed(gen, ild)

```

The loadshed function returns a column vector of MW curtailments of dispatchable loads, computed as the difference between the PG and PMIN values in the corresponding rows of the gen matrix. The optional ild argument is a column vector of generator indices to the dispatchable loads of interest.

\subsection*{9.2.8 printpf}
printpf(results, fd, mpopt)

The printpf function prints power flow and optimal power flow results, as returned to fd, a file identifier which defaults to STDOUT (the screen). The details of what gets printed are controlled by an optional MATPOWER options struct mpopt.

\subsection*{9.2.9 total_load}
```

Pd = total_load(mpc)
Pd = total_load(mpc, load_zone, opt, mpopt)
Pd = total_load(bus)
Pd = total_load(bus, gen, load_zone, opt, mpopt)
[Pd, Qd] = total_load(...)

```

The total_load function returns a vector of total load in each load zone. The opt argument controls whether it includes fixed loads, dispatchable loads or both, and for dispatchable loads, whether to use the nominal or realized load values. The load_zone argument defines the load zones across which loads will be summed. It uses the BUS_AREA column (7) of the bus matrix by default. The string value 'all' can be used to specify a single zone including the entire system. The reactive demands are also optionally available as an output.

\subsection*{9.2.10 totcost}
```

totalcost = totcost(gencost, Pg)

```

The totcost function computes the total cost for generators given a matrix in gencost format and a column vector or matrix of generation levels. The return value has the same dimensions as Pg. Each row of gencost is used to evaluate the cost at the output levels specified in the corresponding row of Pg. The rows of gencost can specify either polynomial or piecewise linear costs and need not be uniform.

\subsection*{9.3 Modifying a Case}

\subsection*{9.3.1 extract_islands}
```

mpc_array = extract_islands(mpc)
mpc_array = extract_islands(mpc, groups)
mpc_k = extract_islands(mpc, k)
mpc_k = extract_islands(mpc, groups, k)
mpc_k = extract_islands(mpc, k, custom)
mpc_k = extract_islands(mpc, groups, k, custom)

```

The extract_islands function extracts individual islands in a network that is not fully connected. The original network is specified as a MATPOWER case struct (mpc) and the result is returned as a cell array of case structs, or as a single case struct. Supplying the optional group avoids the need to traverse the network again, saving
time on large systems. A final optional argument custom is a struct that can be used to indicate custom fields of mpc from which to extract data corresponding to buses generators, branches or DC lines.

\subsection*{9.3.2 load2disp}
```

mpc = load2disp(mpc0)
mpc = load2disp(mpc0, fname)
mpc = load2disp(mpc0, fname, idx)
mpc = load2disp(mpc0, fname, idx, voll)

```

The load2disp function takes a Matpower case mpc0, converts fixed loads to dispatchable loads, curtailable at a specific price, and returns the resulting case struct mpc. It can optionally save the resulting case to a file (fname), convert loads only at specific buses (idx), and set the value of lost load (voll) to be used as the curtailment price (default is \(\$ 5,000 / \mathrm{MWh}\) ).

\subsection*{9.3.3 modcost}
```

newgencost = modcost(gencost, alpha)
newgencost = modcost(gencost, alpha, modtype)

```

The modcost function can be used to modify generator cost functions by shifting or scaling them, either horizontally or vertically. The alpha argument specifies the numerical value of the modification, and modtype defines the type of modification as a string that takes one of the following values: 'SCALE_F' (default), 'SCALE_X', 'SHIFT_F', or 'SHIFT_X'.

\subsection*{9.3.4 scale_load}
```

mpc = scale_load(load, mpc)
mpc = scale_load(load, mpc, load_zone)
mpc = scale_load(load, mpc, load_zone, opt)
bus = scale_load(load, bus)
[bus, gen] = scale_load(load, bus, gen, load_zone, opt)
[bus, gen, gencost] = ...
scale_load(load, bus, gen, load_zone, opt, gencost)

```

The scale_load function is used to scale active (and optionally reactive) loads in each zone by a zone-specific ratio, i.e. \(R(k)\) for zone \(k\). The amount of scaling for
each zone, either as a direct scale factor or as a target quantity, is specified in load. The load zones are defined by load_zone, and opt specifies the type of scaling (factor or target quantity) and which loads are affected (active, reactive or both and fixed, dispatchable or both). The costs (gencost) associated with dispatchable loads can also be optionally scaled with the loads.

\subsection*{9.3.5 apply_changes}
```

mpc_modified = apply_changes(label, mpc_original, chgtab)

```

The apply_changes function implements a general mechanism to apply a set of changes to a base Matpower case. This can be used, for example, to define and apply a set of contingencies. There are three basic types of changes, those that replace old values with new ones, those that scale old values by some factor, and those that add a constant to existing values.

The change table matrix, chgtab, specifies modifications to be applied to an existing case. These modifications are grouped into sets, designated change sets, that are always applied as a group. A change set consists of one or more changes, each specified in a separate row in the chgtab, where the rows share a common label (integer ID).

For example, \(n_{c}\) change sets can be used to define \(n_{c}\) contingencies via a single chgtab with many rows, but only \(n_{c}\) unique labels. The chgtab also optionally specifies a probability \(\pi_{k}\) of occurance associated with change set \(k\). Table 9-1 summarizes the meaning of the data in each column of chgtab. All of the names referenced in Tables 9-1 through 9-4 are defined as constants by the idx_ct function. Type help idx_ct at the Matlab prompt for more details. Use of the named constants when constructing a chgtab matrix is encouraged to improve readability.

The value in the CT_TABLE column of chgtab defines which data table is to be modified and the options are given in Table 9-2. With the exception of load and certain generator cost changes, each individual change record specifies modification(s) to a single column of a particular data matrix, either bus, gen, branch or gencost. Some are changes to that column for an individual row in the matrix (or all rows, if the row index is set to 0 ), while others are area-wide changes that modify all rows corresponding to the specified area. \({ }^{42}\)

Load changes are special and may modify multiple columns of the bus and/or gen tables. They offer a more flexible and convenient means of specifying modifications

\footnotetext{
\({ }^{42}\) Areas are defined by the BUS_AREA column of the bus matrix.
}

Table 9-1: Columns of chgtab
\begin{tabular}{lcl}
\hline name & column & description \\
\hline CT_LABEL & 1 & change set label, unique for each change set (integer) \\
CT_PROB & 2 & \begin{tabular}{l} 
change set probability (number between 0 and 1) \\
type of table to be modified (see Table 9-2 for possible values)
\end{tabular} \\
CT_TABLE & 3 & \begin{tabular}{l} 
tow index of data to be modified, 0 means all rows, \\
CT_ROW
\end{tabular} \\
rT_COL & 5 & \begin{tabular}{l} 
for area-wide changes this is the area index, rather than row index \\
column index of data to be modified (see Table 9-4 for exceptions)
\end{tabular} \\
CT_CHGTYPE & 6 & \begin{tabular}{l} 
type of change, e.g. replace, scale or add (see Table 9-3 for details) \\
CT_NEWVAL
\end{tabular} \\
\hline
\end{tabular}
\(\dagger\) The change set probability \(\pi_{k}\) is taken from this column of the first row for change set \(k\).
to loads (fixed, dispatchable, real and/or reactive) than directly including individual change specifications for each of the corresponding entries in the bus and gen matrices. The row indices for load changes refer to bus numbers.

In addition to the normal direct modifications for generator cost parameters, there is also the option to scale or shift an entire cost function, either vertically or horizontally. This is often more convenient than manipulating the individual cost parameters directly, especially when dealing with a mix of polynomial and piecewise linear generator costs.

Table 9-2: Values for CT_TABLE Column
\begin{tabular}{lcl}
\hline name & value & description \\
\hline CT_TBUS & 1 & bus table \\
CT_TGEN & 2 & gen table \\
CT_TBRCH & 3 & branch table \\
CT_TAREABUS & 4 & area-wide change in bus table \\
CT_TAREAGEN & 5 & area-wide change in gen table \\
CT_TAREABRCH & 6 & area-wide change in branch table \\
CT_TLOAD & 7 & per bus load change \({ }^{\dagger}\) \\
CT_TAREALOAD & 8 & area-wide load change \({ }^{\dagger}\) \\
CT_TGENCOST \(_{\text {CT_TAREAGENCOST }}\) & 9 & gencost table \\
\hline
\end{tabular}
\(\dagger\) Preferred method of modifying load, as opposed to manipulating bus and gen tables directly.

Normally, the CT_COL column contains the column index of the entry or entries in the data tables to be modified. And the CT_CHGTYPE and CT_NEWVAL columns specify, respectively, the type of change (replacement, scaling or adding) and the correspond-
ing replacement value, scale factor or constant to add, as shown in Table 9-3.
Table 9-3: Values for CT_CHGTYPE Column
\begin{tabular}{lcl}
\hline name & value & description \\
\hline CT_REP & 1 & replace old value by new one in CT_NEWVAL column \\
CT_REL & 2 & scale old value by factor in CT_NEWVAL column \\
CT_ADD & 3 & add value in CT_NEWVAL column to old value \\
\hline
\end{tabular}

For load changes, the CT_COL column is not a column index, but rather a code that defines which loads at the specified bus(es) are to be modified, with the ability to select fixed loads only, dispatchable loads only or both, and for each whether or not to include the reactive load in the change. Similarly, the CT_COL column for generator cost modifications can be set to a special code to indicate a scaling or shifting of the entire corresponding cost function(s). The various options for the CT_COL column are summarized in Table 9-4.

Table 9-4: Values for CT_COL Column
\begin{tabular}{ccc}
\hline name & value & description \\
\hline for CT_TABLE column & \(=\) CT_TLOAD or CT_TAREALOAD \\
CT_LOAD_ALL_PQ & 1 & modify all (fixed \& dispatchable) loads, active and reactive \\
CT_LOAD_FIX_PQ & 2 & modify fixed loads, active and reactive \\
CT_LOAD_DIS_PQ & 3 & modify dispatchable loads, active and reactive \\
CT_LOAD_ALL_P & 4 & modify all (fixed \& dispatchable) loads, active power only \\
CT_LOAD_FIX_P & 5 & modify fixed loads, active power only \\
CT_LOAD_DIS_P & 6 & modify dispatchable loads, active power only \\
\hline for CT_TABLE column \(=\) CT_TGENCOST or CT_TAREAGENCOST \\
CT_MODCOST_F & -1 & Scales or shifts the cost function vertically \({ }^{\dagger}\) \\
CT_MODCOST_X & -2 & scales or shifts the cost function horizontally \({ }^{\dagger}\) \\
\hline otherwise \({ }^{\ddagger}\) \\
& \(n\) \\
\hline
\end{tabular}

For example, setting up a chgtab matrix for the following four scenarios could be done as shown below.
1. Turn off generator 2 ( \(10 \%\) probability).
2. Reduce the line rating of all lines to \(95 \%\) of their nominal values ( \(0.2 \%\) probability).
3. Scale all loads in area 2 (real \& reactive, fixed \& dispatchable) up by \(10 \%\) (0.1\% probability).
4. Decrease capacity of generator 3 and shift its cost function to the left both by 10 MW (5\% probability).
```

chgtab = [ ...
1 0.1 CT_TGEN 2 GEN_STATUS CT_REP 0;
2 0.002 CT_TBRCH 0 RATE_A CT_REL 0.95;
3 0.001 CT_TAREALOAD 2 CT_LOAD_ALL_PQ CT_REL 1.1;
4 0.05 CT_TGEN 3 PMAX CT_ADD -10;
4 0.05 CT_TGENCOST 3 CT_MODCOST_X CT_ADD -10;
];

```

A change table can be used to easily create modified cases from an existing base case with the apply_changes function. Given the chgtab from the example above, a new case with all lines derated by \(5 \%\) can easily be created from an existing case mpc with the following line of code.
```

mpc_new = apply_changes(2, mpc, chgtab);

```

\subsection*{9.3.6 savechgtab}
```

savechgtab(fname, chgtab)
savechgtab(fname, chgtab, warnings)
fname = savechgtab(fname, ...)

```

This function can be used to save a change table matrix, chgtab, to a file specified by fname. If the fname string ends with '.mat' it saves chgtab and warnings to a MAT-file as the variables chgtab and warnings, respectively. Otherwise, it saves an M-file function that returns the chgtab, with the optional warnings included in the comments, where warnings is a cell array of warning messages such as those returned by pssecon2chgtab.

\subsection*{9.4 Conversion between External and Internal Numbering}

\subsection*{9.4.1 ext2int, int2ext}
```

mpc_int = ext2int(mpc_ext)
mpc_int = ext2int(mpc_ext, mpopt)
mpc_ext = int2ext(mpc_int)
mpc_ext = int2ext(mpc_int, mpopt)

```

These functions convert a MATPOWER case struct from external to internal, and from internal to external numbering, respectively. ext2int first removes all isolated buses, off-line generators and branches, and any generators or branches connected to isolated buses. Then the buses are renumbered consecutively, beginning at 1 , and the generators are sorted by increasing bus number. Any 'ext2int' callback routines registered in the case are also invoked automatically. All of the related indexing information and the original data matrices are stored in an 'order' field in the struct to be used later by codeint2ext to perform the reverse conversions. If the case is already using internal numbering it is returned unchanged. The optional Matpower options struct (mpopt) input argument is only needed in conjunction with callback routines that depend on it, e.g. when toggle_softlims is on.

\subsection*{9.4.2 e2i_data, i2e_data}
```

val = e2i_data(mpc, val, ordering)
val = e2i_data(mpc, val, ordering, dim)
val = i2e_data(mpc, val, oldval, ordering)
val = i2e_data(mpc, val, oldval, ordering, dim)

```

These functions can be used to convert other data structures from external to internal indexing and vice versa. When given a case struct (mpc) that has already been converted to internal indexing, e2i_data can be used to convert other data structures as well by passing in 2 or 3 extra parameters in addition to the case struct. If the value passed in the second argument (val) is a column vector or cell array, it will be converted according to the ordering specified by the third argument (described below). If val is an \(n\)-dimensional matrix or cell array, then the optional fourth argument (dim, default \(=1\) ) can be used to specify which dimension to reorder. The return value in this case is the value passed in, converted to internal indexing.

The third argument, ordering, is used to indicate whether the data corresponds to bus-, gen- or branch-ordered data. It can be one of the following three strings: 'bus', 'gen' or 'branch'. For data structures with multiple blocks of data, ordered
by bus, gen or branch, they can be converted with a single call by specifying ordering as a cell array of strings.

Any extra elements, rows, columns, etc. beyond those indicated in ordering, are not disturbed.

The function i2e_data performs the opposite conversion, from internal back to external indexing. It also assumes that mpc is using internal indexing, and the only difference is that it also includes an oldval argument used to initialize the return value before converting val to external indexing. In particular, any data corresponding to off-line gens or branches or isolated buses or any connected gens or branches will be taken from oldval, with val supplying the rest of the returned data.

\subsection*{9.4.3 e2i_field, i2e_field}
```

mpc = e2i_field(mpc, field, ordering)
mpc = e2i_field(mpc, field, ordering, dim)
mpc = i2e_field(mpc, field, ordering)
mpc = i2e_field(mpc, field, ordering, dim)

```

These functions can be used to convert additional fields in mpc from external to internal indexing and vice versa. When given a case struct that has already been converted to internal indexing, e2i_field can be used to convert other fields as well by passing in 2 or 3 extra parameters in addition to the case struct.

The second argument (field) is a string or cell array of strings, specifying a field in the case struct whose value should be converted by a corresponding call to e2i_data. The field can contain either a numeric or a cell array. The converted value is stored back in the specified field, the original value is saved for later use and the updated case struct is returned. If field is a cell array of strings, they specify nested fields.

The third and optional fourth arguments (ordering and dim) are simply passed along to the call to e2i_data.

Similarly, i2e_field performs the opposite conversion, from internal back to external indexing. It also assumes that mpc is using internal indexing and utilizes the original data stored by e2i_field, calling i2e_data to do the conversion work.

\subsection*{9.5 Forming Standard Power Systems Matrices}

\subsection*{9.5.1 makeB}
```

[Bp, Bpp] = makeB(mpc, alg)
[Bp, Bpp] = makeB(baseMVA, bus, branch, alg)

```

The makeB function builds the two matrices \(B^{\prime}\) and \(B^{\prime \prime}\) used in the fast-decoupled power flow. The alg, which can take values 'FDXB' or 'FDBX', determines whether the matrices returned correspond to the XB or BX version of the fast-decoupled power flow. Bus numbers must be consecutive beginning at 1 (i.e. internal ordering).

\subsection*{9.5.2 makeBdc}
```

[Bbus, Bf, Pbusinj, Pfinj] = makeBdc(mpc)
[Bbus, Bf, Pbusinj, Pfinj] = makeBdc(baseMVA, bus, branch)

```

The function builds the \(B\) matrices, \(B_{d c}\) (Bbus) and \(B_{f}(\mathrm{Bf})\), and phase shift injections, \(P_{d c}\) (Pbusinj) and \(P_{f, \text { shift }}\) (Pfinj), for the DC power flow model as described in (3.29) and (4.7). Bus numbers must be consecutive beginning at 1 (i.e. internal ordering).

\subsection*{9.5.3 makeJac}
```

J = makeJac(mpc)
J = makeJac(mpc, fullJac)
[J, Ybus, Yf, Yt] = makejac(mpc)

```

The makeJac function forms the power flow Jacobian and, optionally, the system admittance matrices. Bus numbers in the input case must be consecutive beginning at 1 (i.e. internal indexing). If the fullJac argument is present and true, it returns the full Jacobian (sensitivities of all bus injections with respect to all voltage angles and magnitudes) as opposed to the reduced version used in the Newton power flow updates.

\subsection*{9.5.4 makeLODF}
```

PTDF = makePTDF(mpc)
LODF = makeLODF(mpc.branch, PTDF)

```

The makeLODF function forms the DC line outage distribution factor matrix for a given PTDF. The matrix is \(n_{b r} \times n_{b r}\), where \(n_{b r}\) is the number of branches. See Section 4.5 on Linear Shift Factors for more details.

\subsection*{9.5.5 makePTDF}
```

H = makePTDF(mpc)
H = makePTDF(mpc, slack)
H = makePTDF(baseMVA, bus, branch)
H = makePTDF(baseMVA, bus, branch, slack)

```

The makePTDF function returns the DC PTDF matrix for a given choice of slack. The matrix is \(n_{b r} \times n_{b}\), where \(n_{b r}\) is the number of branches and \(n_{b}\) is the number of buses. The slack can be a scalar (single slack bus) or an \(n_{b} \times 1\) column vector of weights specifying the proportion of the slack taken up at each bus. If the slack is not specified, the reference bus is used by default. Bus numbers must be consecutive beginning at 1 (i.e. internal ordering). See Section 4.5 on Linear Shift Factors for more details.

\subsection*{9.5.6 makeYbus}
```

[Ybus, Yf, Yt] = makeYbus(mpc)
[Ybus, Yf, Yt] = makeYbus(baseMVA, bus, branch)

```

The makeYbus function builds the bus admittance matrix and branch admittance matrices from (3.11)-(3.13). Bus numbers in the input case must be consecutive beginning at 1 (i.e. internal indexing).

\subsection*{9.6 Miscellaneous}

\subsection*{9.6.1 define_constants}
```

define_constants

```

The define_constants script is a convenience script that defines a set of useful constants, mostly to be used as named column indices into the bus, branch, gen and gencost data matrices. The purpose is to avoid having to remember column numbers and to allow code to be more robust against potential future changes to the Matpower case data format. It also defines constants for the change tables used by apply_changes.

Specifically, it includes all of the constants defined by idx_bus, idx_brch, idx_gen, idx_cost and idx_ct.

\subsection*{9.6.2 feval_w_path}
```

[y1, ..., yn] = feval_w_path(fpath, f, x1, ..., xn)

```

The feval_w_path function is identical to Matlab's own feval, except that the function \(f\) need not be in the Matlab path if it is defined in a file in the path specified by fpath. Assumes that the current working directory is always first in the Matlab path.

\subsection*{9.6.3 have_fcn}
```

TorF = have_fcn(tag)
TorF = have_fcn(tag, toggle)
ver_str = have_fcn(tag, 'vstr')
ver_num = have_fcn(tag, 'vnum')
rdate = have_fcn(tag, 'date')
info = have_fcn(tag, 'all')

```

The have_fen function provides a unified mechanism for testing for optional functionality, such as the presence of certain solvers, or to detect whether the code is running under Matlab or Octave. Since its results are cached they allow for a very quick way to check frequently for functionality that may initially be a bit more costly to determine. For installed functionality, have_fen also determines the installed version and release date, if possible. The optional second argument, when it is a string, defines which value is returned, as follows:
- empty - 1 if optional functionality is available, 0 if not available
- 'vstr' - version number as a string (e.g. '3.11.4')
- 'vnum' - version number as numeric value (e.g. 3.011004)
- 'date' - release date as a string (e.g. '20-Jan-2015')
- 'all' - struct with fields named av (for "availability"), vstr, vnum and date, and values corresponding to each of the above, respectively.

Alternatively, the optional functionality specified by tag can be toggled OFF or ON by calling have_fcn with a numeric second argument toggle with one of the following values:
- 0 - turn OFF the optional functionality
- 1 - turn ON the optional functionality (if available)
- -1 - toggle the ON/OFF state of the optional functionality

\subsection*{9.6.4 mpopt2qpopt}
```

qpopt = mpopt2qpopt(mpopt)
qpopt = mpopt2qpopt(mpopt, model)
qpopt = mpopt2qpopt(mpopt, model, alg)

```

The mpopt2qpopt function returns an options struct suitable for qps matpower, miqps_matpower or one of the solver specific equivalents. It is constructed from the relevant portions of mpopt, a MATPOWER options struct. The model argument specifies whether the problem to be solved is an LP, QP, MILP or MIQP problem to allow for the selection of a suitable default solver. The final alg argument allows the solver to be set explicitly (in qpopt.alg). By default this value is taken from mpopt.opf.dc.solver.

When the solver is set to 'DEFAULT', this function also selects the best available solver that is applicable \({ }^{43}\) to the specific problem class, based on the following precedence: Gurobi, CPLEX, MOSEK, Optimization Toolbox, GLPK, BPMPD, MIPS.

\footnotetext{
\({ }^{43}\) GLPK is not available for problems with quadratic costs (QP and MIQP), BPMPD and MIPS are not available for mixed integer problems (MILP and MIQP), and the Optimization Toolbox is not an option for problems that combine the two (MIQP).
}

\subsection*{9.6.5 mpver}
```

mpver
v = mpver
v = mpver('all')

```

The mpver function returns the current Matpower version number. With the optional 'all' argument, it returns a struct with the fields 'Name', 'Version', 'Release' and 'Date' (all strings). Calling mpver without assigning the return value prints the version and release date of the current installation of Matpower, Matlab (or Octave), the Optimization Toolbox, MIPS and any optional Matpower packages.

\subsection*{9.6.6 nested_struct_copy}
```

ds = nested_struct_copy(d, s)
ds = nested_struct_copy(d, s, opt)

```

The nested_struct_copy function copies values from a source struct s to a destination struct \(d\) in a nested, recursive manner. That is, the value of each field in \(s\) is copied directly to the corresponding field in d , unless that value is itself a struct, in which case the copy is done via a recursive call to nested_struct_copy. Certain aspects of the copy behavior can be controled via the optional options struct opt, including the possible checking of valid field names.

\section*{10 Acknowledgments}

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\section*{Appendix A MIPS - Matpower Interior Point Solver}

Beginning with version 4, Matpower includes a new primal-dual interior point solver called MIPS, for Matpower Interior Point Solver. It is implemented in pureMatlab code, derived from the MEX implementation of the algorithms described in \([26,41]\).

This solver has application outside of MATPOWER to general nonlinear optimization problems of the following form:
\[
\begin{equation*}
\min _{x} f(x) \tag{A.1}
\end{equation*}
\]
subject to
\[
\begin{gather*}
g(x)=0  \tag{A.2}\\
h(x) \leq 0  \tag{A.3}\\
l \leq A x \leq u  \tag{A.4}\\
x_{\min } \leq x \leq x_{\max } \tag{A.5}
\end{gather*}
\]
where \(f: \mathbb{R}^{n} \rightarrow \mathbb{R}, g: \mathbb{R}^{n} \rightarrow \mathbb{R}^{m}\) and \(h: \mathbb{R}^{n} \rightarrow \mathbb{R}^{p}\).
The solver is implemented by the mips function, which can be called as follows,
```

[x, f, exitflag, output, lambda] = ...
mips(f_fcn, x0, A, l, u, xmin, xmax, gh_fcn, hess_fcn, opt);

```
where the input and output arguments are described in Tables A-1 and A-2, respectively. Alternatively, the input arguments can be packaged as fields in a problem struct and passed in as a single argument, where all fields except \(f \_f c n\) and \(x 0\) are optional.
```

[x, f, exitflag, output, lambda] = mips(problem);

```

The calling syntax is nearly identical to that used by fmincon from Matlab's Optimization Toolbox. The primary difference is that the linear constraints are specified in terms of a single doubly-bounded linear function \((l \leq A x \leq u)\) as opposed to separate equality constrained \(\left(A_{e q} x=b_{e q}\right)\) and upper bounded \((A x \leq b)\) functions. Internally, equality constraints are handled explicitly and determined at run-time based on the values of \(l\) and \(u\).

The user-defined functions for evaluating the objective function, constraints and Hessian are identical to those required by fmincon, with one exception described

Table A-1: Input Arguments for mips \({ }^{\dagger}\)
\begin{tabular}{|c|c|}
\hline name & description \\
\hline f_fon & \begin{tabular}{l}
Handle to a function that evaluates the objective function, its gradients and Hessian \({ }^{\ddagger}\) for a given value of \(x\). Calling syntax for this function: \\
[f, df, d2f] = f_fcn(x)
\end{tabular} \\
\hline x0 & Starting value of optimization vector \(x\). \\
\hline A, \(1, \mathrm{u}\) & Define the optional linear constraints \(l \leq A x \leq u\). Default values for the elements of l and u are -Inf and Inf, respectively. \\
\hline xmin, xmax & Optional lower and upper bounds on the \(x\) variables, defaults are -Inf and Inf, respectively. \\
\hline gh_fen & \begin{tabular}{l}
Handle to function that evaluates the optional nonlinear constraints and their gradients for a given value of \(x\). Calling syntax for this function is: \\
\([h, g, d h, d g]=\) gh_fcn(x)
\end{tabular} \\
\hline hess_fcn & \begin{tabular}{l}
Handle to function that computes the Hessian \({ }^{\ddagger}\) of the Lagrangian for given values of \(x, \lambda\) and \(\mu\), where \(\lambda\) and \(\mu\) are the multipliers on the equality and inequality constraints, \(g\) and \(h\), respectively. The calling syntax for this function is:
\[
\text { Lxx }=\text { hess_fcn(x, lam, cost_mult), }
\] \\
where \(\lambda=\) lam.eqnonlin, \(\mu=\) lam.ineqnonlin and cost_mult is a parameter used to scale the objective function
\end{tabular} \\
\hline opt & Optional options structure with fields, all of which are also optional, described in Table A-3. \\
\hline problem & Alternative, single argument input struct with fields corresponding to arguments above. \\
\hline
\end{tabular}
\(\dagger\) All inputs are optional except f_fen and x0.
\(\ddagger\) If gh_fcn is provided then hess_fcn is also required. Specifically, if there are nonlinear constraints, the Hessian information must provided by the hess_fen function and it need not be computed in f_fen.
below for the Hessian evaluation function. Specifically, \(f \_f\) en should return \(f\) as the scalar objective function value \(f(x)\), df as an \(n \times 1\) vector equal to \(\nabla f\) and, unless gh_fen is provided and the Hessian is computed by hess_fen, d2f as an \(n \times n\) matrix equal to the Hessian \(\frac{\partial^{2} f}{\partial x^{2}}\). Similarly, the constraint evaluation function gh_fcn must return the \(m \times 1\) vector of nonlinear equality constraint violations \(g(x)\), the \(p \times 1\) vector of nonlinear inequality constraint violations \(h(x)\) along with their gradients in dg and dh. Here dg is an \(n \times m\) matrix whose \(j^{\text {th }}\) column is \(\nabla g_{j}\) and dh is \(n \times p\), with \(j^{\text {th }}\) column equal to \(\nabla h_{j}\). Finally, for cases with nonlinear constraints, hess_fcn returns the \(n \times n\) Hessian \(\frac{\partial^{2} \mathcal{L}}{\partial x^{2}}\) of the Lagrangian function
\[
\begin{equation*}
\mathcal{L}(x, \lambda, \mu, \sigma)=\sigma f(x)+\lambda^{\top} g(x)+\mu^{\top} h(x) \tag{A.6}
\end{equation*}
\]
for given values of the multipliers \(\lambda\) and \(\mu\), where \(\sigma\) is the cost_mult scale factor for the objective function. Unlike fmincon, mips passes this scale factor to the Hessian evaluation function in the \(3^{\text {rd }}\) argument.

Table A-2: Output Arguments for mips


The use of nargout in f_fcn and gh_fcn is recommended so that the gradients and Hessian are only computed when required.

\section*{A. 1 Example 1}

The following code shows a simple example of using mips to solve a 2-dimensional unconstrained optimization of Rosenbrock's "banana" function \({ }^{44}\)
\[
\begin{equation*}
f(x)=100\left(x_{2}-x_{1}^{2}\right)^{2}+\left(1-x_{1}\right)^{2} \tag{A.7}
\end{equation*}
\]

First, create a Matlab function that will evaluate the objective function, its gradients and Hessian, for a given value of \(x\). In this case, the coefficient of the first term is defined as a paramter a.

\footnotetext{
\({ }^{44}\) http://en.wikipedia.org/wiki/Rosenbrock_function
}

Table A-3: Options for mips \({ }^{\dagger}\)
\begin{tabular}{|c|c|c|}
\hline name & default & description \\
\hline opt.verbose & 0 & controls level of progress output displayed \\
\hline & & 0 - print no progress info \\
\hline & & 1 - print a little progress info \\
\hline & & 2 - print a lot of progress info \\
\hline & & 3 - print all progress info \\
\hline opt.feastol & \(10^{-6}\) & termination tolerance for feasibility condition \\
\hline opt.gradtol & \(10^{-6}\) & termination tolerance for gradient condition \\
\hline opt.comptol & \(10^{-6}\) & termination tolerance for complementarity condition \\
\hline opt.costtol & \(10^{-6}\) & termination tolerance for cost condition \\
\hline opt.max_it & 150 & maximum number of iterations \\
\hline opt.step_control & 0 & set to 1 to enable step-size control \\
\hline opt.sc.red_it & 20 & max number of step-size reductions if step-control is on \\
\hline opt.costmult & 1 & cost multiplier used to scale the objective function for improved conditioning. Note: This value is also passed as the \(3^{\text {rd }}\) argument to the Hessian evaluation function so that it can appropriately scale the objective function term in the Hessian of the Lagrangian. \\
\hline opt.xi & 0.99995 & \(\xi\) constant used in \(\alpha\) updates in (A.46) and (A.47) \\
\hline opt.sigma & 0.1 & centering parameter \(\sigma\) used in \(\gamma\) update in (A.52) \\
\hline opt.z0 & 1 & used to initialize elements of slack variable \(Z\) \\
\hline opt.alpha_min & \(10^{-8}\) & algorithm returns "Numerically Failed" if the \(\alpha_{p}\) or \(\alpha_{d}\) from (A.46) and (A.47) become smaller than this value \\
\hline opt.rho_min & 0.95 & lower bound on \(\rho_{t}\) corresponding to \(1-\eta\) in Fig. 5 in [26] \\
\hline opt.rho_max & 1.05 & upper bound on \(\rho_{t}\) corresponding to \(1+\eta\) in Fig. 5 in [26] \\
\hline opt.mu_threshold & \(10^{-5}\) & Kuhn-Tucker multipliers smaller than this value for non-binding constraints are forced to zero \\
\hline opt.max_stepsize & \(10^{10}\) & algorithm returns "Numerically Failed" if the 2-norm of the Newton step \(\left[\begin{array}{c}\Delta X \\ \Delta \lambda\end{array}\right]\) from (A.45) exceeds this value \\
\hline
\end{tabular}
```

function [f, df, d2f] = banana(x, a)
f = a*(x(2)-x(1)^2)^2+(1-x(1))^2;
if nargout > 1 %% gradient is required
df = [ 4*a*(x(1) - 3 - x(1)*x(2)) + 2*x(1)-2;
2*a*(x(2) - x(1)^2) ];
if nargout > 2 %% Hessian is required
d2f = 4*a*[ 3*x(1)^2 - x(2) + 1/(2*a), -x(1);
-x(1) 1/2 ];
end
end

```

Then, create a handle to the function, defining the value of the paramter a to be 100 , set up the starting value of \(x\), and call the mips function to solve it.
```

>> f_fcn = @(x)banana(x, 100);
>> x0 = [-1.9; 2];
>> [x, f] = mips(f_fcn, x0)
x =
1
1
f =
0

```

\section*{A. 2 Example 2}

The second example \({ }^{45}\) solves the following 3-dimensional constrained optimization, printing the details of the solver's progress:
\[
\begin{equation*}
\min _{x} f(x)=-x_{1} x_{2}-x_{2} x_{3} \tag{A.8}
\end{equation*}
\]
subject to
\[
\begin{align*}
x_{1}^{2}-x_{2}^{2}+x_{3}^{2}-2 & \leq 0  \tag{A.9}\\
x_{1}^{2}+x_{2}^{2}+x_{3}^{2}-10 & \leq 0 \tag{A.10}
\end{align*}
\]

First, create a Matlab function to evaluate the objective function and its gradients, \({ }^{46}\)

\footnotetext{
\({ }^{45}\) From http://en.wikipedia.org/wiki/Nonlinear_programming\#3-dimensional_example.
\({ }^{46}\) Since the problem has nonlinear constraints and the Hessian is provided by hess_fcn, this function will never be called with three output arguments, so the code to compute d 2 f is actually not necessary.
}
```

function [f, df, d2f] = f2(x)
f = -x(1)*x(2) - x(2)*x(3);
if nargout > 1 %% gradient is required
df = - [x(2); x(1)+x(3); x(2)];
if nargout > 2 %% Hessian is required
d2f = -[0 1 0; 1 0 1; 0 1 0]; %% actually not used since
end %% 'hess_fcn' is provided
end

```
one to evaluate the constraints, in this case inequalities only, and their gradients,
```

function [h, g, dh, dg] = gh2(x)
h = [ 1 -1 1; 1 1 1] * x.^2 + [-2; -10];
dh = 2 * [x(1) x(1); -x(2) x(2); x(3) x(3)];
g = []; dg = [];

```
and another to evaluate the Hessian of the Lagrangian.
```

function Lxx = hess2(x, lam, cost_mult)
if nargin < 3, cost_mult = 1; end %% allows to be used with 'fmincon'
mu = lam.ineqnonlin;
Lxx = cost_mult * [0 -1 0; -1 0 -1; 0 -1 0] + ...
[2*[11 1]*mu 0 0; 0 2*[-1 1]*mu 0; 0 0 2*[[1 1]*mu];

```

Then create a problem struct with handles to these functions, a starting value for \(x\) and an option to print the solver's progress. Finally, pass this struct to mips to solve the problem and print some of the return values to get the output below.
```

function example2
problem = struct( ...
'f_fcn', @(x)f2(x), ...
'gh_fcn', @(x)gh2(x), ...
'hess_fcn', @(x, lam, cost_mult)hess2(x, lam, cost_mult), ...
'x0', [1; 1; 0], ...
'opt', struct('verbose', 2) ...
);
[x, f, exitflag, output, lambda] = mips(problem);
fprintf('\nf = %g exitflag = %d\n', f, exitflag);
fprintf('\nx = \n');
fprintf(' %g\n', x);
fprintf('\nlambda.ineqnonlin =\n');
fprintf(' %g\n', lambda.ineqnonlin);

```
```

>> example2
MATPOWER Interior Point Solver -- MIPS, Version 1.3, 30-Oct-2018
(using built-in linear solver)

| it | objective | step size | feascond | gradcond | compcond | costcond |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: |
| - - | -1 |  | 0 | 1.5 | 5 | 0 |
| 0 | -5.3250167 | 1.6875 | 0 | 0.894235 | 0.850653 | 2.16251 |
| 1 | -7.4708991 | 0.97413 | 0.129183 | 0.00936418 | 0.117278 | 0.339269 |
| 2 | 0 | 0.00174933 | 0.0196518 | 0.0490616 |  |  |
| 3 | -7.0553031 | 0.10406 | 0 | 0.00041301 | 0.0030084 | 0.00165402 |
| 4 | -7.0686267 | 0.034574 | 0 | $1.53531 \mathrm{e}-05$ | 0.000337971 | 0.000245844 |
| 5 | -7.0706104 | 0.0065191 | 0 | $1.22094 \mathrm{e}-07$ | $3.41308 \mathrm{e}-05$ | $4.99387 \mathrm{e}-05$ |
| 6 | -7.0710134 | 0.00062152 | 0 | $9.84879 \mathrm{e}-10$ | $3.41587 \mathrm{e}-06$ | $6.05875 \mathrm{e}-06$ |
| 7 | -7.0710623 | $5.7217 \mathrm{e}-05$ | 0 | $9.73527 \mathrm{e}-12$ | $3.41615 \mathrm{e}-07$ | $6.15483 \mathrm{e}-07$ |
| 8 | -7.0710673 | $5.6761 \mathrm{e}-06$ |  |  |  |  |

Converged!
f = -7.07107 exitflag = 1
x =
1.58114
2.23607
1.58114
lambda.ineqnonlin =
0
0.707107

```

More example problems for mips can be found in t_mips.m.

\section*{A. 3 Quadratic Programming Solver}

A convenience wrapper function called qps_mips is provided to make it trivial to set up and solve linear programming (LP) and quadratic programming (QP) problems of the following form:
\[
\begin{equation*}
\min _{x} \frac{1}{2} x^{\top} H x+c^{\top} x \tag{A.11}
\end{equation*}
\]
subject to
\[
\begin{align*}
l & \leq A x \leq u  \tag{A.12}\\
x_{\min } & \leq x \leq x_{\max } . \tag{A.13}
\end{align*}
\]

Instead of a function handle, the objective function is specified in terms of the paramters \(H\) and \(c\) of quadratic cost coefficients. Internally, qps_mips passes mips
the handle of a function that uses these paramters to evaluate the objective function, gradients and Hessian.

The calling syntax for qps_mips is similar to that used by quadprog from the Matlab Optimization Toolbox.
```

[x, f, exitflag, output, lambda] = qps_mips(H, c, A, l, u, xmin, xmax, x0, opt);

```

Alternatively, the input arguments can be packaged as fields in a problem struct and passed in as a single argument, where all fields except \(\mathrm{H}, \mathrm{c}, \mathrm{A}\) and 1 are optional.
```

[x, f, exitflag, output, lambda] = qps_mips(problem);

```

Aside from H and c, all input and output arguments correspond exactly to the same arguments for mips as described in Tables A-1 and A-2.

As with mips and fmincon, the primary difference between the calling syntax for qps_mips and quadprog is that the linear constraints are specified in terms of a single doubly-bounded linear function \((l \leq A x \leq u)\) as opposed to separate equality constrained \(\left(A_{e q} x=b_{e q}\right)\) and upper bounded \((A x \leq b)\) functions.

MATPOWER also includes another wrapper function qps_matpower that provides a consistent interface for all of the QP and LP solvers it has available. This interface is identical to that used by qps_mips with the exception of the structure of the opt input argument. The solver is chosen according to the value of opt.alg. See the help for qps_matpower for details.

Several examples of using qps_matpower to solve LP and QP problems can be found in t_qps_matpower.m.

\section*{A. 4 Primal-Dual Interior Point Algorithm}

This section provides some details on the primal-dual interior point algorithm used by MIPS and described in \([26,41]\).

\section*{A.4.1 Notation}

For a scalar function \(f: \mathbb{R}^{n} \rightarrow \mathbb{R}\) of a real vector \(X=\left[\begin{array}{llll}x_{1} & x_{2} & \cdots & x_{n}\end{array}\right]^{\top}\), we use the following notation for the first derivatives (transpose of the gradient):
\[
f_{X}=\frac{\partial f}{\partial X}=\left[\begin{array}{llll}
\frac{\partial f}{\partial x_{1}} & \frac{\partial f}{\partial x_{2}} & \cdots & \frac{\partial f}{\partial x_{n}} \tag{A.14}
\end{array}\right] .
\]

The matrix of second partial derivatives, the Hessian of \(f\), is:
\[
f_{X X}=\frac{\partial^{2} f}{\partial X^{2}}=\frac{\partial}{\partial X}\left(\frac{\partial f}{\partial X}\right)^{\top}=\left[\begin{array}{ccc}
\frac{\partial^{2} f}{\partial x_{1}^{2}} & \cdots & \frac{\partial^{2} f}{\partial x_{1} \partial x_{n}}  \tag{A.15}\\
\vdots & \ddots & \vdots \\
\frac{\partial^{2} f}{\partial x_{n} \partial x_{1}} & \cdots & \frac{\partial^{2} f}{\partial x_{n}^{2}}
\end{array}\right]
\]

For a vector function \(F: \mathbb{R}^{n} \rightarrow \mathbb{R}^{m}\) of a vector \(X\), where
\[
F(X)=\left[\begin{array}{llll}
f_{1}(X) & f_{2}(X) & \cdots & f_{m}(X) \tag{A.16}
\end{array}\right]^{\top}
\]
the first derivatives form the Jacobian matrix, where row \(i\) is the transpose of the gradient of \(f_{i}\)
\[
F_{X}=\frac{\partial F}{\partial X}=\left[\begin{array}{ccc}
\frac{\partial f_{1}}{\partial x_{1}} & \cdots & \frac{\partial f_{1}}{\partial x_{n}}  \tag{A.17}\\
\vdots & \ddots & \vdots \\
\frac{\partial f_{m}}{\partial x_{1}} & \cdots & \frac{\partial f_{m}}{\partial x_{n}}
\end{array}\right]
\]

In these derivations, the full 3-dimensional set of second partial derivatives of \(F\) will not be computed. Instead a matrix of partial derivatives will be formed by computing the Jacobian of the vector function obtained by multiplying the transpose of the Jacobian of \(F\) by a vector \(\lambda\), using the following notation
\[
\begin{equation*}
F_{X X}(\lambda)=\frac{\partial}{\partial X}\left(F_{X}^{\top} \lambda\right) . \tag{A.18}
\end{equation*}
\]

Please note also that \([A]\) is used to denote a diagonal matrix with vector \(A\) on the diagonal and \(e\) is a vector of all ones.

\section*{A.4.2 Problem Formulation and Lagrangian}

The primal-dual interior point method used by MIPS solves a problem of the form:
\[
\begin{equation*}
\min _{X} f(X) \tag{A.19}
\end{equation*}
\]
subject to
\[
\begin{align*}
& G(X)=0  \tag{A.20}\\
& H(X) \leq 0 \tag{A.21}
\end{align*}
\]
where the linear constraints and variable bounds from (A.4) and (A.5) have been incorporated into \(G(X)\) and \(H(X)\). The approach taken involves converting the \(n_{i}\)
inequality constraints into equality constraints using a barrier function and vector of positive slack variables \(Z\).
\[
\begin{equation*}
\min _{X}\left[f(X)-\gamma \sum_{m=1}^{n_{i}} \ln \left(Z_{m}\right)\right] \tag{A.22}
\end{equation*}
\]
subject to
\[
\begin{align*}
G(X) & =0  \tag{A.23}\\
H(X)+Z & =0  \tag{A.24}\\
Z & >0 \tag{A.25}
\end{align*}
\]

As the parameter of perturbation \(\gamma\) approaches zero, the solution to this problem approaches that of the original problem.

For a given value of \(\gamma\), the Lagrangian for this equality constrained problem is
\[
\begin{equation*}
\mathcal{L}^{\gamma}(X, Z, \lambda, \mu)=f(X)+\lambda^{\top} G(X)+\mu^{\top}(H(X)+Z)-\gamma \sum_{m=1}^{n_{i}} \ln \left(Z_{m}\right) \tag{A.26}
\end{equation*}
\]

Taking the partial derivatives with respect to each of the variables yields:
\[
\begin{align*}
\mathcal{L}_{X}^{\gamma}(X, Z, \lambda, \mu) & =f_{X}+\lambda^{\top} G_{X}+\mu^{\top} H_{X}  \tag{A.27}\\
\mathcal{L}_{Z}^{\gamma}(X, Z, \lambda, \mu) & =\mu^{\top}-\gamma e^{\top}[Z]^{-1}  \tag{A.28}\\
\mathcal{L}_{\lambda}^{\gamma}(X, Z, \lambda, \mu) & =G^{\top}(X)  \tag{A.29}\\
\mathcal{L}_{\mu}^{\gamma}(X, Z, \lambda, \mu) & =H^{\top}(X)+Z^{\top} \tag{A.30}
\end{align*}
\]

And the Hessian of the Lagrangian with respect to \(X\) is given by
\[
\begin{equation*}
\mathcal{L}_{X X}^{\gamma}(X, Z, \lambda, \mu)=f_{X X}+G_{X X}(\lambda)+H_{X X}(\mu) \tag{A.31}
\end{equation*}
\]

\section*{A.4.3 First Order Optimality Conditions}

The first order optimality (Karush-Kuhn-Tucker) conditions for this problem are satisfied when the partial derivatives of the Lagrangian above are all set to zero:
\[
\begin{align*}
F(X, Z, \lambda, \mu) & =0  \tag{A.32}\\
Z & >0  \tag{A.33}\\
\mu & >0 \tag{A.34}
\end{align*}
\]
where
\[
F(X, Z, \lambda, \mu)=\left[\begin{array}{c}
\mathcal{L}_{X}^{\gamma}{ }^{\top}  \tag{A.35}\\
{[\mu] Z-\gamma e} \\
G(X) \\
H(X)+Z
\end{array}\right]=\left[\begin{array}{c}
f_{X}^{\top}+G_{X}^{\top} \lambda+H_{X}{ }^{\top} \mu \\
{[\mu] Z-\gamma e} \\
G(X) \\
H(X)+Z
\end{array}\right]
\]

\section*{A.4.4 Newton Step}

The first order optimality conditions are solved using Newton's method. The Newton update step can be written as follows:
\[
\begin{align*}
{\left[\begin{array}{llll}
F_{X} & F_{Z} & F_{\lambda} & F_{\mu}
\end{array}\right]\left[\begin{array}{c}
\Delta X \\
\Delta Z \\
\Delta \lambda \\
\Delta \mu
\end{array}\right] } & =-F(X, Z, \lambda, \mu)  \tag{A.36}\\
{\left[\begin{array}{cccc}
\mathcal{L}_{X X}^{\gamma} & 0 & G_{X}{ }^{\top} & H_{X}^{\top} \\
0 & {[\mu]} & 0 & {[Z]} \\
G_{X} & 0 & 0 & 0 \\
H_{X} & I & 0 & 0
\end{array}\right]\left[\begin{array}{c}
\Delta X \\
\Delta Z \\
\Delta \lambda \\
\Delta \mu
\end{array}\right] } & =-\left[\begin{array}{c}
\mathcal{L}_{X}^{\gamma} \\
{[\mu] Z-\gamma e} \\
G(X) \\
H(X)+Z
\end{array}\right] . \tag{A.37}
\end{align*}
\]

This set of equations can be simplified and reduced to a smaller set of equations by solving explicitly for \(\Delta \mu\) in terms of \(\Delta Z\) and for \(\Delta Z\) in terms of \(\Delta X\). Taking the \(2^{\text {nd }}\) row of (A.37) and solving for \(\Delta \mu\) we get
\[
\begin{align*}
{[\mu] \Delta Z+[Z] \Delta \mu } & =-[\mu] Z+\gamma e \\
{[Z] \Delta \mu } & =-[Z] \mu+\gamma e-[\mu] \Delta Z \\
\Delta \mu & =-\mu+[Z]^{-1}(\gamma e-[\mu] \Delta Z) \tag{A.38}
\end{align*}
\]

Solving the \(4^{\text {th }}\) row of (A.37) for \(\Delta Z\) yields
\[
\begin{align*}
H_{X} \Delta X+\Delta Z & =-H(X)-Z \\
\Delta Z & =-H(X)-Z-H_{X} \Delta X . \tag{A.39}
\end{align*}
\]

Then, substituting (A.38) and (A.39) into the \(1^{\text {st }}\) row of (A.37) results in
\[
\begin{array}{ll}
\mathcal{L}_{X X}^{\gamma} \Delta X+G_{X}^{\top} \Delta \lambda+H_{X}^{\top} \Delta \mu & =-\mathcal{L}_{X}^{\gamma}{ }^{\top} \\
\mathcal{L}_{X X}^{\gamma} \Delta X+G_{X}^{\top} \Delta \lambda+H_{X}^{\top}\left(-\mu+[Z]^{-1}(\gamma e-[\mu] \Delta Z)\right) & =-\mathcal{L}_{X}^{\gamma}{ }^{\top} \\
\mathcal{L}_{X X}^{\gamma} \Delta X+G_{X}^{\top} \Delta \lambda & =-\mathcal{L}_{X}^{\gamma}{ }^{\top} \\
+H_{X}^{\top}\left(-\mu+[Z]^{-1}\left(\gamma e-[\mu]\left(-H(X)-Z-H_{X} \Delta X\right)\right)\right) & \\
\mathcal{L}_{X X}^{\gamma} \Delta X+G_{X}^{\top} \Delta \lambda-H_{X}^{\top} \mu+H_{X}^{\top}[Z]^{-1} \gamma e & =-\mathcal{L}_{X}^{\gamma}{ }^{\top} \\
+H_{X}^{\top}[Z]^{-1}[\mu] H(X)+H_{X}^{\top}[Z]^{-1}[Z] \mu+H_{X}^{\top}[Z]^{-1}[\mu] H_{X} \Delta X & \\
\left(\mathcal{L}_{X X}^{\gamma}+H_{X}^{\top}[Z]^{-1}[\mu] H_{X}\right) \Delta X+G_{X}^{\top} \Delta \lambda & =-\mathcal{L}_{X}^{\gamma}{ }^{\top} \\
+H_{X}^{\top}[Z]^{-1}(\gamma e+[\mu] H(X)) & =-N \\
M \Delta X+G_{X}^{\top} \Delta \lambda & \tag{A.40}
\end{array}
\]
where
\[
\begin{align*}
M & \equiv \mathcal{L}_{X X}^{\gamma}+H_{X}^{\top}[Z]^{-1}[\mu] H_{X}  \tag{A.41}\\
& =f_{X X}+G_{X X}(\lambda)+H_{X X}(\mu)+H_{X}^{\top}[Z]^{-1}[\mu] H_{X} \tag{A.42}
\end{align*}
\]
and
\[
\begin{align*}
N & \equiv \mathcal{L}_{X}^{\gamma}{ }^{\top}+H_{X}^{\top}[Z]^{-1}(\gamma e+[\mu] H(X))  \tag{A.43}\\
& =f_{X}^{\top}+G_{X}^{\top} \lambda+H_{X}^{\top} \mu+H_{X}^{\top}[Z]^{-1}(\gamma e+[\mu] H(X)) \tag{A.44}
\end{align*}
\]

Combining (A.40) and the \(3^{\text {rd }}\) row of (A.37) results in a system of equations of reduced size:
\[
\left[\begin{array}{cc}
M & G_{X}^{\top}  \tag{A.45}\\
G_{X} & 0
\end{array}\right]\left[\begin{array}{c}
\Delta X \\
\Delta \lambda
\end{array}\right]=\left[\begin{array}{c}
-N \\
-G(X)
\end{array}\right]
\]

The Newton update can then be computed in the following 3 steps:
1. Compute \(\Delta X\) and \(\Delta \lambda\) from (A.45).
2. Compute \(\Delta Z\) from (A.39).
3. Compute \(\Delta \mu\) from (A.38).

In order to maintain strict feasibility of the trial solution, the algorithm truncates the Newton step by scaling the primal and dual variables by \(\alpha_{p}\) and \(\alpha_{d}\), respectively,
where these scale factors are computed as follows:
\[
\begin{align*}
& \alpha_{p}=\min \left(\xi \min _{\Delta Z_{m}<0}\left(-\frac{Z_{m}}{\Delta Z_{m}}\right), 1\right)  \tag{A.46}\\
& \alpha_{d}=\min \left(\xi \min _{\Delta \mu_{m}<0}\left(-\frac{\mu_{m}}{\Delta \mu_{m}}\right), 1\right) \tag{A.47}
\end{align*}
\]
resulting in the variable updates below.
\[
\begin{gather*}
X \leftarrow X+\alpha_{p} \Delta X  \tag{A.48}\\
Z \leftarrow Z+\alpha_{p} \Delta Z  \tag{A.49}\\
\lambda \leftarrow \lambda+\alpha_{d} \Delta \lambda  \tag{A.50}\\
\mu \leftarrow \mu+\alpha_{d} \Delta \mu \tag{A.51}
\end{gather*}
\]

The parameter \(\xi\) is a constant scalar with a value slightly less than one. In MIPS, \(\xi\) is set to 0.99995 .

In this method, during the Newton-like iterations, the perturbation parameter \(\gamma\) must converge to zero in order to satisfy the first order optimality conditions of the original problem. MIPS uses the following rule to update \(\gamma\) at each iteration, after updating \(Z\) and \(\mu\) :
\[
\begin{equation*}
\gamma \leftarrow \sigma \frac{Z^{\top} \mu}{n_{i}} \tag{A.52}
\end{equation*}
\]
where \(\sigma\) is a scalar constant between 0 and 1 . In MIPS, \(\sigma\) is set to 0.1 .

\section*{Appendix B Data File Format}

There are two versions of the Matpower case file format. Matpower versions 3.0.0 and earlier used the version 1 format internally. Subsequent versions of Matpower have used the version 2 format described below, though version 1 files are still handled, and converted automatically, by the loadcase and savecase functions.

In the version 2 format, the input data for Matpower are specified in a set of data matrices packaged as the fields of a Matlab struct, referred to as a "MatPOWER case" struct and conventionally denoted by the variable mpc. This struct is typically defined in a case file, either a function M-file whose return value is the mpc struct or a MAT-file that defines a variable named mpc when loaded. The fields of this struct are baseMVA, bus, branch, gen and, optionally, gencost. The baseMVA field is a scalar and the rest are matrices. Each row in the data matrices corresponds to a single bus, branch, or generator and the columns are similar to the columns in the standard IEEE and PTI formats. The mpc struct also has a version field whose value is a string set to the current Matpower case version, currently '2' by default. The version 1 case format defines the data matrices as individual variables rather than fields of a struct, and some do not include all of the columns defined in version 2.

Numerous examples can be found in the case files listed in Table D-18 in Appendix D. The case files created by savecase use a tab-delimited format for the data matrices to make it simple to transfer data seamlessly back and forth between a text editor and a spreadsheet via simple copy and paste.

The details of the Matpower case format are given in the tables below and can also be accessed by typing help caseformat at the Matlab prompt. First, the baseMVA field is a simple scalar value specifying the system MVA base used for converting power into per unit quantities. For convenience and code portability, idx_bus defines a set of constants to be used as named indices into the columns of the bus matrix. Similarly, idx_brch, idx_gen and idx_cost define names for the columns of branch, gen and gencost, respectively. The script define_constants provides a simple way to define all the usual constants at one shot. These are the names that appear in the first column of the tables below.

The Matpower case format also allows for additional fields to be included in the structure. The OPF is designed to recognize fields named A, \(1, u, H, C w, N, f p a r m, z 0\), zl and zu as parameters used to directly extend the OPF formulation as described in Section 7.1. Additional standard optional fields include bus_name, gentype and genfuel. \({ }^{47}\) Other user-defined fields may also be included, such as the reserves field used in the example code throughout Section 7.3. The loadcase function will automatically load any extra fields from a case file and, if the appropriate 'savecase' callback function (see Section 7.3.5) is added via add_userfcn, savecase will also save them back to a case file.

Table B-1: Bus Data (mpc.bus)
\begin{tabular}{lcl}
\hline name & column & description \\
\hline BUS_I & 1 & bus number (positive integer) \\
BUS_TYPE & 2 & bus type (1=PQ, \(2=\mathrm{PV}, 3=\) ref, \(4=\) isolated) \\
PD & 3 & real power demand (MW) \\
QD & 4 & reactive power demand (MVAr) \\
GS & 5 & shunt conductance (MW demanded at \(V=1.0\) p.u.) \\
BS & 6 & shunt susceptance (MVAr injected at \(V=1.0\) p.u.) \\
BUS_AREA & 7 & area number (positive integer) \\
VM & 8 & voltage magnitude (p.u.) \\
VA & 9 & voltage angle (degrees) \\
BASE_KV & 10 & base voltage (kV) \\
ZONE & 11 & loss zone (positive integer) \\
VMAX \(_{\text {VMIN }}\) & 12 & maximum voltage magnitude (p.u.) \\
LAM_P \({ }^{\dagger}\) & 13 & minimum voltage magnitude (p.u.) \\
LAM_Q \(^{\dagger}\) & 14 & Lagrange multiplier on real power mismatch \((u / \mathrm{MW})\) \\
MU_VMAX \(^{\dagger}\) & 15 & Lagrange multiplier on reactive power mismatch \((u / \mathrm{MVAr})\) \\
MU_VMIN \(^{\dagger}\) & 16 & Kuhn-Tucker multiplier on upper voltage limit \((u / \mathrm{p} . \mathrm{u})\). \\
\hline
\end{tabular}
\({ }^{\dagger}\) Included in OPF output, typically not included (or ignored) in input matrix. Here we assume the objective function has units \(u\).

\footnotetext{
\({ }^{47}\) All three of these are cell arrays of strings. See gentypes and genfuels for more information on the corresponding fields.
}

Table B-2: Generator Data (mpc.gen)
\begin{tabular}{|c|c|c|}
\hline name & column & description \\
\hline GEN_BUS & 1 & bus number \\
\hline PG & 2 & real power output (MW) \\
\hline QG & 3 & reactive power output (MVAr) \\
\hline QMAX & 4 & maximum reactive power output (MVAr) \\
\hline QMIN & 5 & minimum reactive power output (MVAr) \\
\hline VG \({ }^{\ddagger}\) & 6 & voltage magnitude setpoint (p.u.) \\
\hline MBASE & 7 & total MVA base of machine, defaults to baseMVA \\
\hline GEN_STATUS & 8 & \begin{tabular}{l}
machine status, \\
\(>0=\) machine in-service \\
\(\leq 0=\) machine out-of-service
\end{tabular} \\
\hline PMAX & 9 & maximum real power output (MW) \\
\hline PMIN & 10 & minimum real power output (MW) \\
\hline PC1* & 11 & lower real power output of PQ capability curve (MW) \\
\hline PC2* & 12 & upper real power output of PQ capability curve (MW) \\
\hline QC1MIN* & 13 & minimum reactive power output at PC1 (MVAr) \\
\hline QC1MAX* & 14 & maximum reactive power output at PC1 (MVAr) \\
\hline QC2MIn* & 15 & minimum reactive power output at PC2 (MVAr) \\
\hline QC2MAX* & 16 & maximum reactive power output at PC2 (MVAr) \\
\hline RAMP_AGC* & 17 & ramp rate for load following/AGC (MW/min) \\
\hline RAMP_10* & 18 & ramp rate for 10 minute reserves (MW) \\
\hline RAMP_30* & 19 & ramp rate for 30 minute reserves (MW) \\
\hline RAMP_Q* & 20 & ramp rate for reactive power ( 2 sec timescale) ( \(\mathrm{MVAr} / \mathrm{min}\) ) \\
\hline APF* & 21 & area participation factor \\
\hline MU_PMAX \({ }^{\dagger}\) & 22 & Kuhn-Tucker multiplier on upper \(P_{g}\) limit ( \(u / \mathrm{MW}\) ) \\
\hline MU_PMIN \({ }^{\dagger}\) & 23 & Kuhn-Tucker multiplier on lower \(P_{g}\) limit ( \(u / \mathrm{MW}\) ) \\
\hline MU_QMAX \({ }^{\dagger}\) & 24 & Kuhn-Tucker multiplier on upper \(Q_{g}\) limit ( \(u / \mathrm{MVAr}\) ) \\
\hline MU_QMIN \({ }^{\dagger}\) & 25 & Kuhn-Tucker multiplier on lower \(Q_{g}\) limit ( \(u / \mathrm{MVAr}\) ) \\
\hline
\end{tabular}

\footnotetext{
* Not included in version 1 case format.
\(\dagger\) Included in OPF output, typically not included (or ignored) in input matrix. Here we assume the objective function has units \(u\).
\(\ddagger\) Used to determine voltage setpoint for optimal power flow only if opf.use_vg option is non-zero ( 0 by default). Otherwise generator voltage range is determined by limits set for corresponding bus in bus matrix.
}

Table B-3: Branch Data (mpc.branch)
\begin{tabular}{|c|c|c|}
\hline name & column & description \\
\hline F_BUS & 1 & "from" bus number \\
\hline T_BUS & 2 & "to" bus number \\
\hline BR_R & 3 & resistance (p.u.) \\
\hline BR_X & 4 & reactance (p.u.) \\
\hline BR_B & 5 & total line charging susceptance (p.u.) \\
\hline RATE_A & 6 & MVA rating A (long term rating), set to 0 for unlimited \\
\hline RATE_B & 7 & MVA rating B (short term rating), set to 0 for unlimited \\
\hline RATE_C & 8 & MVA rating C (emergency rating), set to 0 for unlimited \\
\hline TAP & 9 & transformer off nominal turns ratio, if non-zero (taps at "from" bus, impedance at "to" bus, i.e. if \(r=x=b=0, \operatorname{tap}=\frac{\left|V_{f}\right|}{\left|V_{t}\right|}\); \(t a p=0\) used to indicate transmission line rather than transformer, i.e. mathematically equivalent to transformer with tap \(=1\) ) \\
\hline SHIFT & 10 & transformer phase shift angle (degrees), positive \(\Rightarrow\) delay \\
\hline BR_STATUS & 11 & initial branch status, \(1=\) in-service, \(0=\) out-of-service \\
\hline ANGMIN* & 12 & minimum angle difference, \(\theta_{f}-\theta_{t}\) (degrees) \\
\hline Angmax* & 13 & maximum angle difference, \(\theta_{f}-\theta_{t}\) (degrees) \\
\hline \(\mathrm{PF}^{\dagger}\) & 14 & real power injected at "from" bus end (MW) \\
\hline QF \({ }^{\dagger}\) & 15 & reactive power injected at "from" bus end (MVAr) \\
\hline \(\mathrm{PT}^{\dagger}\) & 16 & real power injected at "to" bus end (MW) \\
\hline QT \({ }^{\dagger}\) & 17 & reactive power injected at "to" bus end (MVAr) \\
\hline MU_SF \({ }^{\ddagger}\) & 18 & Kuhn-Tucker multiplier on MVA limit at "from" bus (u/MVA) \\
\hline MU_ST \({ }^{\ddagger}\) & 19 & Kuhn-Tucker multiplier on MVA limit at "to" bus ( \(u / \mathrm{MVA}\) ) \\
\hline MU_ANGMIN \({ }^{\ddagger}\) & 20 & Kuhn-Tucker multiplier lower angle difference limit (u/degree) \\
\hline MU_ANGMAX \({ }^{\ddagger}\) & 21 & Kuhn-Tucker multiplier upper angle difference limit ( \(u\) /degree) \\
\hline
\end{tabular}

\footnotetext{
* Not included in version 1 case format. The voltage angle difference is taken to be unbounded below if ANGMIN \(\leq-360\) and unbounded above if ANGMAX \(\geq 360\). If both parameters are zero, the voltage angle difference is unconstrained.
\(\dagger\) Included in power flow and OPF output, ignored on input.
\(\ddagger\) Included in OPF output, typically not included (or ignored) in input matrix. Here we assume the objective function has units \(u\).
}

Table B-4: Generator Cost Data \({ }^{\dagger}\) (mpc.gencost)
\begin{tabular}{|c|c|c|}
\hline name & column & description \\
\hline MODEL & 1 & cost model, \(1=\) piecewise linear, \(2=\) polynomial \\
\hline STARTUP & 2 & startup cost in US dollars* \\
\hline SHUTDOWN & 3 & shutdown cost in US dollars* \\
\hline NCOST & 4 & number of cost coefficients for polynomial cost function, or number of data points for piecewise linear \\
\hline COST & 5 & \begin{tabular}{l}
parameters defining total cost function \(f(p)\) begin in this column, units of \(f\) and \(p\) are \(\$ / \mathrm{hr}\) and MW (or MVAr), respectively \\
\((\) MODEL \(=1) \Rightarrow \quad p_{0}, f_{0}, p_{1}, f_{1}, \ldots, p_{n}, f_{n}\) \\
where \(p_{0}<p_{1}<\cdots<p_{n}\) and the cost \(f(p)\) is defined by the coordinates \(\left(p_{0}, f_{0}\right),\left(p_{1}, f_{1}\right), \ldots,\left(p_{n}, f_{n}\right)\) \\
of the end/break-points of the piecewise linear cost \\
\((\) MODEL \(=2) \Rightarrow \quad c_{n}, \ldots, c_{1}, c_{0}\) \\
\(n+1\) coefficients of \(n\)-th order polynomial cost, starting with highest order, where cost is \(f(p)=c_{n} p^{n}+\cdots+c_{1} p+c_{0}\)
\end{tabular} \\
\hline
\end{tabular}
\(\dagger\) If gen has \(n_{g}\) rows, then the first \(n_{g}\) rows of gencost contain the costs for active power produced by the corresponding generators. If gencost has \(2 n_{g}\) rows, then rows \(n_{g}+1\) through \(2 n_{g}\) contain the reactive power costs in the same format.
* Not currently used by any Matpower functions.

Table B-5: DC Line Data* (mpc.dcline)
\begin{tabular}{|c|c|c|}
\hline name & column & description \\
\hline F_BUS & 1 & "from" bus number \\
\hline T_BUS & 2 & "to" bus number \\
\hline BR_STATUS & 3 & initial branch status, \(1=\) in-service, \(0=\) out-of-service \\
\hline \(\mathrm{PF}^{\dagger}\) & 4 & real power flow at "from" bus end (MW), "from" \(\rightarrow\) "to" \\
\hline \(\mathrm{PT}^{\dagger}\) & 5 & real power flow at "to" bus end (MW), "from" \(\rightarrow\) "to" \\
\hline Q \({ }^{\dagger}\) & 6 & reactive power injected into "from" bus (MVAr) \\
\hline QT \({ }^{\dagger}\) & 7 & reactive power injected into "to" bus (MVAr) \\
\hline VF & 8 & voltage magnitude setpoint at "from" bus (p.u.) \\
\hline VT & 9 & voltage magnitude setpoint at "to" bus (p.u.) \\
\hline PMIN & 10 & if positive (negative), lower limit on PF (PT) \\
\hline PMAX & 11 & if positive (negative), upper limit on PF (PT) \\
\hline QMINF & 12 & lower limit on reactive power injection into "from" bus (MVAr) \\
\hline QMAXF & 13 & upper limit on reactive power injection into "from" bus (MVAr) \\
\hline QMINT & 14 & lower limit on reactive power injection into "to" bus (MVAr) \\
\hline QMAXT & 15 & upper limit on reactive power injection into "to" bus (MVAr) \\
\hline LOSSO & 16 & coefficient \(l_{0}\) of constant term of linear loss function (MW) \\
\hline LOSS1 & 17 & coefficient \(l_{1}\) of linear term of linear loss function (MW/MW) ( \(p_{\text {loss }}=l_{0}+l_{1} p_{f}\), where \(p_{f}\) is the flow at the "from" end) \\
\hline MU_PMIN \({ }^{\ddagger}\) & 18 & Kuhn-Tucker multiplier on lower flow limit at "from" bus (u/MW) \\
\hline MU_PMAX \({ }^{\ddagger}\) & 19 & Kuhn-Tucker multiplier on upper flow limit at "from" bus (u/MW) \\
\hline MU_QMINF \({ }^{\ddagger}\) & 20 & Kuhn-Tucker multiplier on lower VAr limit at "from" bus (u/MVAr) \\
\hline MU_QMAXF \({ }^{\ddagger}\) & 21 & Kuhn-Tucker multiplier on upper VAr limit at "from" bus (u/MVAr) \\
\hline MU_QMINT \({ }^{\ddagger}\) & 22 & Kuhn-Tucker multiplier on lower VAr limit at "to" bus (u/MVAr) \\
\hline MU_QMAXT \({ }^{\ddagger}\) & 23 & Kuhn-Tucker multiplier on upper VAr limit at "to" bus (u/MVAr) \\
\hline
\end{tabular}

\footnotetext{
* Requires explicit use of toggle_dcline.
\(\dagger\) Output column, value updated by power flow or OPF (except PF in case of simple power flow).
\(\ddagger\) Included in OPF output, typically not included (or ignored) in input matrix. Here we assume the objective function has units \(u\).
}

\section*{Appendix C Matpower Options}

Beginning with version 4.2, Matpower uses an options struct to control the many options available. Earlier versions used an options vector with named elements. MATPOWER's options are used to control things such as:
- power flow algorithm
- power flow termination criterion
- power flow options (e.g. enforcing of reactive power generation limits)
- continuation power flow options
- OPF algorithm
- OPF termination criterion
- OPF options (e.g. active vs. apparent power vs. current for line limits)
- verbose level
- printing of results
- solver specific options

As with the old-style options vector, the options struct should always be created and modified using the mpoption function to ensure compatibility across different versions of Matpower. The default Matpower options struct is obtained by calling mpoption with no arguments.
```

>> mpopt = mpoption;

```

Individual options can be overridden from their default values by calling mpoption with a set of name/value pairs as input arguments. For example, the following runs a fast-decoupled power flow of case30 with very verbose progress output:
```

>> mpopt = mpoption('pf.alg', 'FDXB', 'verbose', 3);
>> runpf('case30', mpopt);

```

For backward compatibility, old-style option names/values can also be used.
```

>> mpopt = mpoption('PF_ALG', 2, 'VERBOSE', 3);

```

Another way to specify option overrides is via a struct. Using the example above, the code would be as follows.
```

>> overrides = struct('pf', struct('alg', 'FDXB'), 'verbose', 3);
>> mpopt = mpoption(overrides);

```

Finally, a string containing the name of a function that returns such a struct, can be passed to mpoption instead of the struct itself.
```

>> mpopt = mpoption('verbose_fast_decoupled_pf_opts');

```
where the function verbose_fast_decoupled_pf_opts is defined as follows:
```

function ov = verbose_fast_decoupled_pf_opts()
ov = struct('pf', struct('alg', 'FDXB'), 'verbose', 3);

```

To make changes to an existing options struct (as opposed to the default options struct), simply include it as the first argument. For example, to modify the previous run to enforce reactive power limts, suppress the pretty-printing of the output and save the results to a struct instead:
```

>> mpopt = mpoption(mpopt, 'pf.enforce_q_lims', 1, 'out.all', 0);
>> results = runpf('case30', mpopt);

```

This works when specifying the overrides as a struct or function name as well. For backward compatibility, the first argument can be an old-style options vector, followed by old-style option name/value pairs.

The available options and their default values are summarized in the following tables and can also be accessed via the command help mpoption. Some of the options require separately installed optional packages available from the MATPOWER website.

Table C-1: Top-Level Options
\begin{tabular}{|c|c|c|}
\hline name & default & description \\
\hline model & ' AC ' & AC vs. DC modeling for power flow and OPF formulation 'AC' - use AC formulation and corresponding algs/options 'DC' - use DC formulation and corresponding algs/options \\
\hline pf & see Table C-2 & power flow options \\
\hline cpf & see Table C-3 & continuation power flow options \\
\hline opf & see Tables C-4, C-5 & optimal power flow options \\
\hline verbose & 1 & amount of progress info printed \\
\hline & & 0 - print no progress info \\
\hline & & 1 - print a little progress info \\
\hline & & 2 - print a lot of progress info \\
\hline & & 3 - print all progress info \\
\hline out & see Table C-6 & pretty-printed output options \\
\hline mips & see Table C-7 & MIPS options \\
\hline clp & see Table C-8 & CLP options* \\
\hline cplex & see Table C-9 & CPLEX options* \\
\hline fmincon & see Table C-10 & fmincon options \({ }^{\dagger}\) \\
\hline glpk & see Table C-11 & GLPK options* \\
\hline gurobi & see Table C-12 & Gurobi options* \\
\hline ipopt & see Table C-13 & IPOPT options* \\
\hline knitro & see Table C-14 & KNITRO options* \\
\hline minopf & see Table C-15 & MINOPF options* \\
\hline mosek & see Table C-16 & MOSEK options* \\
\hline pdipm & see Table C-17 & PDIPM options** \\
\hline tralm & see Table C-18 & TRALM options* \\
\hline
\end{tabular}

\footnotetext{
\({ }^{*}\) Requires the installation of an optional package. See Appendix G for details on the corresponding package.
\(\dagger\) Requires Matlab's Optimization Toolbox, available from The MathWorks, Inc (http://www.mathworks.com/).
}

Table C-2: Power Flow Options
\begin{tabular}{|c|c|c|}
\hline name & default & description \\
\hline \multirow[t]{8}{*}{pf.alg} & \multirow[t]{8}{*}{' NR '} & AC power flow algorithm: \\
\hline & & 'NR' - Newtons's method \\
\hline & & 'FDXB' - Fast-Decoupled (XB version) \\
\hline & & 'FDBX' - Fast-Decouple (BX version) \\
\hline & & 'GS' - Gauss-Seidel \\
\hline & & 'PQSUM ' - Power Summation (radial networks only) \\
\hline & & 'ISUM' - Current Summation (radial networks only) \\
\hline & & 'YSUM' - Admittance Summation (radial networks only) \\
\hline pf.tol & \(10^{-8}\) & termination tolerance on per unit P and Q dispatch \\
\hline pf.nr.max_it & 10 & maximum number of iterations for Newton's method \\
\hline \multirow[t]{5}{*}{pf.nr.lin_solver} & \multirow[t]{4}{*}{''} & linear solver option for mplinsolve for computing Newton update step (see mplinsolve for complete list of all options) \\
\hline & & \begin{tabular}{l}
'' - default to '\' for small systems, 'LU3' for larger ones \\
' \' - built-in backslash operator
\end{tabular} \\
\hline & & 'LU' - explicit default LU decomposition and back substitution \\
\hline & & \begin{tabular}{l}
'LU3' - 3 output arg form of 1 u , Gilbert-Peierls algorithm with approximate minimum degree (AMD) reordering \\
'LU4' - 4 output arg form of 1 u, UMFPACK solver (same as 'LU')
\end{tabular} \\
\hline & 30 & 'LU5' - 5 output arg form of 1 u , UMFPACK solver w/row scaling maximum number of iterations for fast-decoupled method \\
\hline pf.fd.max_it & 30 & maximum number of iterations for fast-decoupled method \\
\hline pf.gs.max_it & 1000 & maximum number of iterations for Gauss-Seidel method \\
\hline pf.radial.max_it & 20 & maximum number of iterations for radial power flow methods \\
\hline pf.radial.vcorr & 0 & \begin{tabular}{l}
perform voltage correction procedure in distribution power flow 0 - do not perform voltage correction \\
1 - perform voltage correction
\end{tabular} \\
\hline pf.enforce_q_lims & 0 & \begin{tabular}{l}
enforce gen reactive power limits at expense of \(\left|V_{m}\right|\) \\
0 - do not enforce limits \\
1 - enforce limits, simultaneous bus type conversion \\
2 - enforce limits, one-at-a-time bus type conversion
\end{tabular} \\
\hline
\end{tabular}

Table C-3: Continuation Power Flow Options
\begin{tabular}{|c|c|c|}
\hline name & default & description \\
\hline \multirow[t]{4}{*}{cpf.parameterization} & 3 & choice of parameterization \\
\hline & & 1 - natural \\
\hline & & 2 - arc length \\
\hline & & 3 - pseudo arc length \\
\hline \multirow[t]{3}{*}{cpf.stop_at} & ' NOSE ' & determines stopping criterion \\
\hline & & 'NOSE' - stop when nose point is reached \\
\hline & & ' FULL' - trace full nose curve \\
\hline \multirow[t]{3}{*}{cpf.enforce_p_lims} & 0 & \(\lambda_{\text {stop }}\) - stop upon reaching target \(\lambda\) value \(\lambda_{\text {stop }}\) enforce gen active power limits \\
\hline & & 0 - do not enforce limits \\
\hline & & 1 - enforce limits \\
\hline \multirow[t]{2}{*}{cpf.enforce_q_lims} & 0 & enforce gen reactive power limits at expense of \(V_{m}\) 0 - do not enforce limits \\
\hline & & 1 - enforce limits \\
\hline \multirow[t]{3}{*}{cpf.enforce_v_lims} & 0 & enforce bus voltage magnitude limits \\
\hline & & 0 - do not enforce limits \\
\hline & & 1 - enforce limits \\
\hline \multirow[t]{3}{*}{cpf.enforce_flow_lims} & 0 & enforce branch MVA flow limits \\
\hline & & \(0-\) do not enforce limits \\
\hline & & 1 - enforce limits \\
\hline cpf.step & 0.05 & default value for continuation power flow step size \(\sigma\) \\
\hline cpf.step_min & \(10^{-4}\) & minimum allowed step size, \(\sigma_{\text {min }}\) \\
\hline cpf.step_max & 0.2 & maximum allowed step size, \(\sigma_{\text {max }}\) \\
\hline \multirow[t]{3}{*}{cpf.adapt_step} & 0 & toggle adaptive step size feature \\
\hline & & 0 - adaptive step size disabled \\
\hline & & 1 - adaptive step size enabled \\
\hline cpf.adapt_step_damping & 0.7 & damping factor \(\beta_{\mathrm{cpf}}\) from (5.13) for adaptive step sizing \\
\hline cpf.adapt_step_tol & \(10^{-3}\) & tolerance \(\epsilon_{\text {cpf }}\) from (5.13) for adaptive step sizing \\
\hline cpf.target_lam_tol & \(10^{-5}\) & tolerance for target lambda detection \\
\hline cpf.nose_tol & \(10^{-5}\) & tolerance for nose point detection (p.u.) \\
\hline cpf.p_lims_tol & \(10^{-2}\) & tolerance for generator active power limit detection (MW) \\
\hline cpf.q_lims_tol & \(10^{-2}\) & tolerance for generator reactive power limit detection (MVAr) \\
\hline cpf.v_lims_tol & \(10^{-4}\) & tolerance for bus voltage magnitude limit detection (p.u) \\
\hline cpf.flow_lims_tol & 0.01 & tolerance for branch flow limit detection (MVA) \\
\hline \multirow[t]{5}{*}{cpf.plot.level} & 0 & control plotting of nose curve \\
\hline & & 0 - do not plot nose curve \\
\hline & & 1 - plot when completed \\
\hline & & 2 - plot incrementally at each iteration \\
\hline & & 3 - same as 2 , with pause at each iteration \\
\hline cpf.plot.bus & empty & index of bus whose voltage is to be plotted \\
\hline cpf.user_callback & empty & string or cell array of strings with names of user callback functions \({ }^{\dagger}\) \\
\hline
\end{tabular}

\footnotetext{
\(\dagger\) See help cpf_default_callback for details.
154
}

Table C-4: OPF Solver Options
\begin{tabular}{|c|c|c|c|}
\hline name & default & description & \\
\hline \multirow[t]{10}{*}{opf.ac.solver} & \multirow[t]{10}{*}{'DEFAULT'} & AC optimal & power flow solver: \\
\hline & & 'DEFAULT' & ' - choose default solver, i.e. 'MIPS' \\
\hline & & 'MIPS' & - MIPS, Matpower Interior Point Solver, primal/dual interior point method \({ }^{\dagger}\) \\
\hline & & 'FMINCON' & ' - Matlab Optimization Toolbox, fmincon \\
\hline & & 'IPOPT' & - IPOPT* \\
\hline & & ' KNITRO' & - KNITRO* \\
\hline & & 'MINOPF' & - MINOPF*, MINOS-based solver \\
\hline & & 'PDIPM' & - PDIPM, primal/dual interior point method \({ }^{\ddagger}\) \\
\hline & & 'SDPOPF' & - SDPOPF, , solver based on semidefinite relaxation \\
\hline & & 'TRALM' & - TRALM, trust region based augmented Langrangian method \\
\hline \multirow[t]{10}{*}{opf.dc.solver} & \multirow[t]{10}{*}{'DEFAULT'} & \multicolumn{2}{|l|}{\begin{tabular}{l}
DC optimal power flow solver: \\
'DEFAULT' - choose default solver based on availability in the following order: 'GUROBI', 'CPLEX', \\
'MOSEK', 'OT', 'GLPK' (linear costs only), \\
'BPMPD', 'MIPS'
\end{tabular}} \\
\hline & & 'MIPS' & - MIPS, Matpower Interior Point Solver, primal/dual interior point method \({ }^{\dagger}\) \\
\hline & & 'BPMPD' & - BPMPD* \\
\hline & & ' CLP' & - CLP* \\
\hline & & 'CPLEX' & - CPLEX* \\
\hline & & ' GLPK ' & - GLPK \({ }^{*}\) (no quadratic costs) \\
\hline & & 'GUROBI' & - Gurobi* \({ }^{*}\) \\
\hline & & 'IPOPT' & - IPOPT* \\
\hline & & ' MOSEK \({ }^{\prime}\) & - MOSEK* \\
\hline & & 'OT' & - Matlab Opt Toolbox, quadprog, linprog \\
\hline
\end{tabular}

\footnotetext{
* Requires the installation of an optional package. See Appendix G for details on the corresponding package.
\(\dagger\) For MIPS-sc, the step-controlled version of this solver, the mips.step_control option must be set to 1.
\(\ddagger\) For SC-PDIPM, the step-controlled version of this solver, the pdipm. step_control option must be set to 1 .
}

Table C-5: General OPF Options
\begin{tabular}{lcc}
\hline name & default & description \\
opf. current_balance & 0 & use current, as opposed to power, balance formulation for \\
opf.v_cartesian & & AC OPF, 0 or 1
\end{tabular}

\footnotetext{
* Deprecated. Use opf.start instead.
\(\dagger\) Using a value between 0 and 1 results in the limits being determined by the corresponding weighted average of the 2 options.
\(\ddagger\) Only applies to fmincon, Ipopt, KNITRO and MIPS solvers, which use an interior point estimate; others use current state in Matpower case, as with opf.start \(=2\).
}

Table C-6: Power Flow and OPF Output Options
\begin{tabular}{|c|c|c|}
\hline name & default & description \\
\hline \multirow[t]{5}{*}{verbose} & \multirow[t]{5}{*}{1} & amount of progress info to be printed \\
\hline & & 0 - print no progress info \\
\hline & & 1 - print a little progress info \\
\hline & & 2 - print a lot of progress info \\
\hline & & 3 - print all progress info \\
\hline \multirow[t]{4}{*}{out.all} & \multirow[t]{4}{*}{-1} & controls pretty-printing of results \\
\hline & & -1 - individual flags control what is printed \\
\hline & & 0 - do not print anything \({ }^{\dagger^{*}}\) \\
\hline & & 1 - print everything \({ }^{\dagger}\) \\
\hline out.sys_sum & 1 & print system summary (0 or 1) \\
\hline out.area_sum & 0 & print area summaries (0 or 1) \\
\hline out.bus & 1 & print bus detail, includes per bus gen info (0 or 1 ) \\
\hline out.branch & 1 & print branch detail (0 or 1) \\
\hline out.gen & 0 & print generator detail (0 or 1) \\
\hline \multirow[t]{5}{*}{out.lim.all} & \multirow[t]{5}{*}{-1} & controls constraint info output \\
\hline & & -1-individual flags control what is printed \\
\hline & & \(0-\) do not print any constraint info \({ }^{\dagger}\) \\
\hline & & 1 - print only binding constraint info \({ }^{\dagger}\) \\
\hline & & \(2-\) print all constraint info \({ }^{\dagger}\) \\
\hline \multirow[t]{4}{*}{out.lim.v} & \multirow[t]{4}{*}{1} & control output of voltage limit info \\
\hline & & \(0-\) do not print \\
\hline & & 1 - print binding constraints only \\
\hline & & 2 - print all constraints \\
\hline out.lim.line & 1 & control output of line flow limit info \({ }^{\ddagger}\) \\
\hline out.lim.pg & 1 & control output of gen active power limit info \({ }^{\ddagger}\) \\
\hline out.lim.qg & 1 & control output of gen reactive power limit info \({ }^{\ddagger}\) \\
\hline out.force & 0 & print results even if success flag \(=0\) (0 or 1) \\
\hline \multirow[t]{4}{*}{out.suppress_detail} & \multirow[t]{4}{*}{-1} & suppress all output but system summary \\
\hline & & -1 - suppress details for large systems ( \(>500\) buses) \\
\hline & & \begin{tabular}{l}
0 - do not suppress any output specified by other flags \\
1 - suppress all output except system summary section \({ }^{\dagger}\)
\end{tabular} \\
\hline & & 1 - suppress all output except system summary section \({ }^{\dagger}\) \\
\hline
\end{tabular}

\footnotetext{
* This setting is ignored for pretty-printed output to files specified as FNAME argument in calls to runpf, runopf, etc.
\(\dagger\) Overrides individual flags, but (in the case of out.suppress_detail) not out.all \(=1\).
\(\ddagger\) Takes values of 0,1 or 2 as for out.lim.v.
}

Table C-7: OPF Options for MIPS
\begin{tabular}{|c|c|c|}
\hline name & default & description \\
\hline mips.linsolver & '' & \[
\begin{aligned}
& \text { linear system solver for update step } \\
& \text { ' ' or '\' - built-in backslash \operator } \\
& \text { 'PARDISO' - PARDISO solver }{ }^{\dagger}
\end{aligned}
\] \\
\hline mips.feastol & 0 & feasibiliy (equality) tolerance set to value of opf.violation by default \\
\hline mips.gradtol & \(10^{-6}\) & gradient tolerance \\
\hline mips.comptol & \(10^{-6}\) & complementarity condition (inequality) tolerance \\
\hline mips.costtol & \(10^{-6}\) & optimality tolerance \\
\hline mips.max_it & 150 & maximum number of iterations \\
\hline mips.step_control & 0 & set to 1 to enable step-size control \\
\hline mips.sc.red_it \({ }^{\ddagger}\) & 20 & maximum number of step size reductions per iteration \\
\hline
\end{tabular}

Table C-8: OPF Options for CLP \({ }^{\dagger}\)
\begin{tabular}{lll}
\hline name & default \(^{\S}\) & description \\
\hline clp.opts & empty & \begin{tabular}{l} 
struct of native CLP options passed to clp_options to override \\
defaults, applied after overrides from clp.opt_fname \\
clp.opt_fname
\end{tabular} \\
empty & \begin{tabular}{l} 
name of user-supplied function passed as FNAME argument to \\
clp_options to override defaults
\end{tabular} \\
\hline
\end{tabular}
\({ }^{\dagger}\) For opf.dc.solver option set to 'CLP' only. Requires the installation of the optional CLP package. See Appendix G. 2 for details.
\(\ddagger\) For details, see help clp_options or help clp.

Table C-9: OPF Options for CPLEX \({ }^{\dagger}\)
\begin{tabular}{|c|c|c|}
\hline name & default & description \\
\hline \multirow[t]{8}{*}{cplex.lpmethod} & \multirow[t]{8}{*}{0} & algorithm used by CPLEX for LP problems \\
\hline & & 0 - automatic; let CPLEX choose \\
\hline & & 1 - primal simplex \\
\hline & & 2 - dual simplex \\
\hline & & 3 - network simplex \\
\hline & & 4 - barrier \\
\hline & & 5 - sifting \\
\hline & & 6 - concurrent (dual, barrier, and primal) \\
\hline \multirow[t]{6}{*}{cplex.qpmethod} & \multirow[t]{6}{*}{0} & algorithm used by CPLEX for QP problems \\
\hline & & 0 - automatic; let CPLEX choose \\
\hline & & 1 - primal simplex \\
\hline & & 2 - dual simplex \\
\hline & & 3 - network simplex \\
\hline & & 4 - barrier \\
\hline cplex.opts & empty & struct of native CPLEX options (for cplexoptimset) passed to cplex_options to override defaults, applied after overrides from cplex.opt_fname \({ }^{\ddagger}\) \\
\hline cplex.opt_fname & empty & name of user-supplied function passed as FNAME argument to cplex_options to override defaults \({ }^{\ddagger}\) \\
\hline cplex.opt & 0 & if cplex.opt_fname is empty and cplex.opt is non-zero, the value of cplex.opt_fname is generated by appending cplex.opt to 'cplex_user_options_' (for backward compatibility with old MATPOWER option CPLEX_OPT) \\
\hline
\end{tabular}
\(\dagger\) For opf.dc.solver option set to 'CPLEX' only. Requires the installation of the optional CPLEX package. See Appendix G. 3 for details.
\(\ddagger\) For details, see help cplex_options and the "Parameters of CPLEX" section of the CPLEX documentation at http://www.ibm.com/support/knowledgecenter/SSSA5P.

Table C-10: OPF Options for fmincon \({ }^{\dagger}\)
\begin{tabular}{|c|c|c|}
\hline name & default & description \\
\hline \multirow[t]{6}{*}{fmincon.alg} & \multirow[t]{6}{*}{4} & algorithm used by fmincon in Matlab Opt Toolbox \(\geq 4\) 1 - active-set \({ }^{\ddagger}\) \\
\hline & & 2 - interior-point, default "bfgs" Hessian approximation \\
\hline & & 3 - interior-point, "lbfgs" Hessian approximation \\
\hline & & 4 - interior-point, exact user-supplied Hessian \\
\hline & & 5 - interior-point, Hessian via finite-differences \\
\hline & & \(6-\mathrm{sqp}\), sequential quadratic programming \({ }^{\ddagger}\) \\
\hline fmincon.tol_x & \(10^{-4}\) & termination tolerance on \(x^{*}\) \\
\hline fmincon.tol_f & \(10^{-4}\) & termination tolerance on \(f^{*}\) \\
\hline fmincon.max_it & 0 & maximum number of iterations* \\
\hline & & \(0 \Rightarrow\) use solver's default value \\
\hline
\end{tabular}

\footnotetext{
\(\dagger\) Requires Matlab's Optimization Toolbox, available from The MathWorks, Inc (http://www.mathworks. com/).
\(\ddagger\) Does not use sparse matrices, so not applicable for large-scale systems.
* Display is set by verbose, TolCon by opf.violation, TolX by fmincon.tol_x, TolFun by fmincon.tol_f, and MaxIter and MaxFunEvals by fmincon.max_it.
}

Table C-11: OPF Options for GLPK \({ }^{\dagger}\)
\begin{tabular}{lcl}
\hline name & default \(^{\S}\) & description \\
\hline glpk.opts & empty & \begin{tabular}{l} 
struct of native GLPK options passed to glpk_options to over- \\
ride defaults, applied after overrides from glpk.opt_fname \\
glpk. opt_fname
\end{tabular} \\
empty & \begin{tabular}{l} 
name of user-supplied function passed as FNAME argument to \\
glpk_options to override defaults
\end{tabular} \\
\hline
\end{tabular}

\footnotetext{
\(\dagger\) For opf.dc.solver option set to 'GLPK' only. Requires the installation of the optional GLPK package. See Appendix G. 4 for details.
\(\ddagger\) For details, see help glpk_options or the "param" section of the GLPK documentation at http://www.gnu. org/software/octave/doc/interpreter/Linear-Programming.html.
}

Table C-12: OPF Options for Gurobi \({ }^{\dagger}\)
\begin{tabular}{|c|c|c|}
\hline name & default§ & description \\
\hline gurobi.method & 0 & algorithm used by Gurobi for LP/QP problems \\
\hline & & 0 - primal simplex \\
\hline & & 1 - dual simplex \\
\hline & & 2 - barrier \\
\hline & & 3 - concurrent (LP only) \\
\hline & & 4 - deterministic concurrent (LP only) \\
\hline gurobi.timelimit & \(\infty\) & maximum time allowed for solver (secs) \\
\hline gurobi.threads & 0 (auto) & maximum number of threads to use \\
\hline gurobi.opts & empty & struct of native Gurobi options passed to gurobi_options to override defaults, applied after overrides from gurobi.opt_fname \({ }^{\ddagger}\) \\
\hline gurobi.opt_fname & empty & name of user-supplied function passed as FNAME argument to gurobi_options to override defaults \({ }^{\ddagger}\) \\
\hline gurobi.opt & 0 & if gurobi.opt_fname is empty and gurobi.opt is non-zero, the value of gurobi.opt_fname is generated by appending gurobi.opt to 'gurobi_user_options_' (for backward compatibility with old Matpower option GRB_OPT) \\
\hline
\end{tabular}
\(\dagger\) For opf.dc.solver option set to 'GUROBI' only. Requires the installation of the optional Gurobi package. See Appendix G. 5 for details.
\(\S\) Default values in parenthesis refer to defaults assigned by Gurobi if called with option equal to 0.
\(\ddagger\) For details, see help gurobi_options and the "Parameters" section of the "Gurobi Optimizer Reference Manual" at http://www.gurobi.com/documentation/6.0/refman/parameters.html.

Table C-13: OPF Options for IPOPT \({ }^{\dagger}\)
\begin{tabular}{lcl}
\hline name & default & description \\
\hline ipopt.opts & empty & \begin{tabular}{l} 
struct of native IPOPT options (options.ipopt for ipopt) \\
passed to ipopt_options to override defaults, applied after over- \\
rides from ipopt.opt_fname \\
ipopt.opt_fname
\end{tabular} \\
ipopt.opt & \(0 \quad\)\begin{tabular}{l} 
name of user-supplied function passed as FNAME argument to \\
ipopt_options to override defaults
\end{tabular} \\
if ipopt.opt_fname is empty and ipopt.opt is non-zero, the \\
value of ipopt.opt_fname is generated by appending ipopt.opt \\
to 'ipopt_user_options_' (for backward compatibility with old \\
MATPOWER option IPOPT_OPT)
\end{tabular}
\(\dagger\) For opf.ac.solver or opf.dc.solver option set to 'IPOPT' only. Requires the installation of the optional Ipopt package [49]. See Appendix G. 6 for details.
\(\ddagger\) For details, see help ipopt_options and the options reference section of the IPOPT documentation at http: //www.coin-or.org/Ipopt/documentation/.

Table C-14: OPF Options for KNITRO \({ }^{\dagger}\)
\begin{tabular}{lcl}
\hline name & default & description \\
\hline knitro.tol_x & \(10^{-4}\) & termination tolerance on \(x\) \\
knitro.tol_f & \(10^{-4}\) & \begin{tabular}{l} 
termination tolerance on \(f\) \\
knitro.maxit
\end{tabular} \\
mnitro.opt_fname & empty & \begin{tabular}{l}
\(0 \Rightarrow\) use solver's default value \\
name of user-supplied native KNITRO options file that overrides \\
other options
\end{tabular} \\
knitro.opt & 0 & \begin{tabular}{l} 
if knitro.opt_fname is empty and knitro.opt is a posi- \\
tive integer \(n\), the value of knitro.opt_fname is generated \\
as 'knitro_user_options_ \(n . t x t ' ~(f o r ~ b a c k w a r d ~ c o m p a t i b i l i t y ~\)
\end{tabular} \\
with old MATPOWER option KNITRO_OPT)
\end{tabular}

\footnotetext{
\(\dagger\) For opf.ac.solver option set to 'KNITRO' only. Requires the installation of the optional KNITRO package [32]. See Appendix G. 7 for details.
\(\ddagger\) Note that KNITRO uses the opt_fname option slightly differently from other optional solvers. Specifically, it is the name of a text file processed directly by KNITRO, not a MATLAB function that returns an options struct passed to the solver.
}

Table C-15: OPF Options for MINOPF \({ }^{\dagger}\)
\begin{tabular}{|c|c|c|}
\hline name & default \({ }^{\ddagger}\) & description \\
\hline minopf.feastol & \(0\left(10^{-3}\right)\) & primal feasibility tolerance set to value of opf.violation by default \\
\hline minopf.rowtol & \(0\left(10^{-3}\right)\) & row tolerance set to value of opf.violation by default \\
\hline minopf.xtol & \(0\left(10^{-4}\right)\) & \(x\) tolerance \\
\hline minopf.majdamp & 0 (0.5) & major damping parameter \\
\hline minopf.mindamp & 0 (2.0) & minor damping parameter \\
\hline minopf.penalty & 0 (1.0) & penalty parameter \\
\hline minopf.major_it & 0 (200) & major iterations \\
\hline minopf.minor_it & 0 (2500) & minor iterations \\
\hline minopf.max_it & 0 (2500) & iteration limit \\
\hline minopf.verbosity & -1 & \begin{tabular}{l}
amount of progress output printed by MEX file \\
-1 - controlled by verbose option \\
0 - do not print anything \\
1 - print only only termination status message \\
2 - print termination status \& screen progress \\
3 - print screen progress, report file (usually fort.9)
\end{tabular} \\
\hline minopf.core & 0 & memory allocation defaults to \(1200 n_{b}+2\left(n_{b}+n_{g}\right)^{2}\) \\
\hline minopf.supbasic_lim minopf.mult_price & \[
\begin{gathered}
0 \\
0(30)
\end{gathered}
\] & superbasics limit, defaults to \(2 n_{b}+2 n_{g}\) multiple price \\
\hline
\end{tabular}

\footnotetext{
\(\dagger\) For opf.ac.solver option set to 'MINOPF' only. Requires the installation of the optional MINOPF package [30]. See Appendix G. 8 for details.
\(\ddagger\) Default values in parenthesis refer to defaults assigned in MEX file if called with option equal to 0 .
}

Table C-16: OPF Options for MOSEK \({ }^{\dagger}\)
\begin{tabular}{|c|c|c|}
\hline name & default \({ }^{\text {§ }}\) & description \\
\hline mosek.lp_alg & 0 & \begin{tabular}{l}
solution algorithm used by MOSEK for continuous LP problems (MSK_IPAR_OPTIMIZER)* \\
0 - automatic; let MOSEK choose \\
1 - interior point \\
3 - primal simplex \\
4 - dual simplex \\
5 - primal dual simplex \\
6 - automatic simplex (MOSEK chooses which simplex) \\
7 - network primal simplex \\
10 - concurrent
\end{tabular} \\
\hline mosek.max_it & 0 (400) & interior point maximum iterations (MSK_IPAR_INTPNT_MAX_ITERATIONS) \\
\hline mosek.gap_tol & \(0\left(10^{-8}\right)\) & interior point relative gap tolerance (MSK_DPAR_INTPNT_TOL_REL_GAP) \\
\hline mosek.max_time & 0 (-1) & \begin{tabular}{l}
maximum time allowed for solver (negative means \(\infty\) ) \\
(MSK_DPAR_OPTIMIZER_MAX_TIME)
\end{tabular} \\
\hline mosek.num_threads & 0 (1) & \begin{tabular}{l}
maximum number of threads to use \\
(MSK_IPAR_INTPNT_NUM_THREADS)
\end{tabular} \\
\hline mosek.opts & empty & struct of native MOSEK options (param struct normally passed to mosekopt) passed to mosek_options to override defaults, applied after overrides from mosek.opt_fname \({ }^{\ddagger}\) \\
\hline mosek.opt_fname & empty & name of user-supplied function passed as FNAME argument to mosek_options to override defaults \({ }^{\ddagger}\) \\
\hline mosek.opt & 0 & if mosek.opt_fname is empty and mosek.opt is non-zero, the value of mosek.opt_fname is generated by appending mosek.opt to 'mosek_user_options_' (for backward compatibility with old MATPOWER option MOSEK_OPT) \\
\hline
\end{tabular}

\footnotetext{
\(\dagger\) For opf.dc.solver option set to 'MOSEK' only. Requires the installation of the optional MOSEK package. See Appendix G. 9 for details.
\(\S\) Default values in parenthesis refer to defaults assigned by MOSEK if called with option equal to 0.
* The values listed here correspond to those used by MOSEK version 7.x. Version 6.x was different. It is probably safer to write your code using the symbolic constants defined by mosek_symbcon rather than using explicit numerical values.
\(\ddagger\) For details, see help mosek_options and the "Parameters" reference in "The MOSEK optimization toolbox for MATLAB manual" at http://docs.mosek.com/7.1/toolbox/Parameters.html.
}

Table C-17: OPF Options for PDIPM \({ }^{\dagger}\)
\begin{tabular}{|c|c|c|}
\hline name & default & description \\
\hline pdipm.feastol & 0 & feasibiliy (equality) tolerance set to value of opf.violation by default \\
\hline pdipm.gradtol & \(10^{-6}\) & gradient tolerance \\
\hline pdipm.comptol & \(10^{-6}\) & complementarity condition (inequality) tolerance \\
\hline pdipm.costtol & \(10^{-6}\) & optimality tolerance \\
\hline pdipm.max_it & 150 & maximum number of iterations \\
\hline pdipm.step_control & 0 & set to 1 to enable step-size control \\
\hline pdipm.sc.red_it \({ }^{\ddagger}\) & 20 & maximum number of step size reductions per iteration \\
\hline pdipm.sc.smooth_ratio \({ }^{\ddagger}\) & 0.04 & piecewise linear curve smoothing ratio \\
\hline
\end{tabular}

Table C-18: OPF Options for TRALM \({ }^{\dagger}\)
\begin{tabular}{lcl}
\hline name & default & description \\
\hline tralm.feastol & 0 & feasibiliy tolerance \\
tralm.primaltol & \(5 \times 10^{-4}\) & \begin{tabular}{l} 
set to value of opf.violation by default \\
tralm.dualtol
\end{tabular} \\
triable tolerance \\
tralm.costtol & \(5 \times 10^{-4}\) & dual variable tolerance \\
tralm.major_it & 40 & optimality tolerance \\
tralm.minor_it & 100 & maximum number of major iterations \\
tralm.smooth_ratio & 0.04 & piecewise linear curve smoothing ratio \\
\hline
\end{tabular}

\footnotetext{
\({ }^{\dagger}\) Requires installation of the optional TSPOPF package [25]. See Appendix G. 13 for details.
}

\section*{C. 1 Mapping of Old-Style Options to New-Style Options}

A MATPOWER options struct can be created from an old-style options vector simply by passing it to mpoption. The mapping of old-style options into the fields in the option struct are summarized in Table C-19. An old-style options vector can also be created from an options struct by calling mpoption with the struct and an empty second argument.
```

mpopt_struct = mpoption(mpopt_vector);
mpopt_vector = mpoption(mpopt_struct, []);

```

Table C-19: Old-Style to New-Style Option Mapping

continued on next page

Table C-19: Old-Style to New-Style Option Mapping - continued


Table C-19: Old-Style to New-Style Option Mapping - continued
\begin{tabular}{|c|c|c|c|}
\hline idx & old option & new option & notes \\
\hline 68 & MNS_MINOR_IT & minopf.minor_it & \\
\hline 69 & MNS_MAX_IT & minopf.max_it & \\
\hline 70 & MNS_VERBOSITY & minopf.verbosity & \\
\hline 71 & MNS_CORE & minopf.core & \\
\hline 72 & MNS_SUPBASIC_LIM & minopf.supbasic_lim & \\
\hline 73 & MNS_MULT_PRICE & minopf.mult_price & \\
\hline 80 & FORCE_PC_EQ_P0 & sopf.force_Pc_eq_P0 & for c3sopf ( not part of MATPOWER) \\
\hline 81 & PDIPM_FEASTOL & mips.feastol, pdipm.feastol & \\
\hline 82 & PDIPM_GRADTOL & mips.gradtol, pdipm.gradtol & \\
\hline 83 & PDIPM_COMPTOL & mips.comptol, pdipm. comptol & \\
\hline 84 & PDIPM_COSTTOL & mips.costtol, pdipm.costtol & \\
\hline 85 & PDIPM_MAX_IT & mips.max_it, pdipm.max_it & \\
\hline 86 & SCPDIPM_RED_IT & mips.sc.red_it, pdipm.sc.red_it & \\
\hline 87 & TRALM_FEASTOL & tralm.feastol & \\
\hline 88 & TRALM_PRIMETOL & tralm.primaltol & \\
\hline 89 & TRALM_DUALTOL & tralm.dualtol & \\
\hline 90 & TRALM_COSTTOL & tralm.costtol & \\
\hline 91 & TRALM_MAJOR_IT & tralm.major_it & \\
\hline 92 & TRALM_MINOR_IT & tralm.minor_it & \\
\hline 93 & SMOOTHING_RATIO & tralm.smooth_ratio, pdipm.sc.smooth_ratio & \\
\hline 95 & CPLEX_LPMETHOD & cplex.lpmethod & \\
\hline 98 & CPLEX_QPMETHOD & cplex.qpmethod & \\
\hline 97 & CPLEX_OPT & cplex.opt & \\
\hline 111 & MOSEK_LP_ALG & mosek.lp_alg & \\
\hline 112 & MOSEK_MAX_IT & mosek.max_it & \\
\hline 113 & MOSEK_GAP_TOL & mosek.gap_tol & \\
\hline 114 & MOSEK_MAX_TIME & mosek.max_time & \\
\hline 115 & MOSEK_NUM_THREADS & mosek.num_threads & \\
\hline 116 & MOSEK_OPT & mosek.opt & \\
\hline
\end{tabular}

Table C-19: Old-Style to New-Style Option Mapping - continued
\begin{tabular}{llll}
\hline idx & old option & new option & notes \\
\hline 121 & GRB_METHOD & gurobi.method & \\
122 & GRB_TIMELIMIT & gurobi.timelimit & \\
123 & GRB_THREADS & gurobi.threads & \\
124 & GRB_OPT & gurobi.opt & \\
\hline
\end{tabular}

\section*{Appendix D Matpower Files and Functions}

This appendix lists all of the files and functions that MATPOWER provides, with the exception of those in the extras directory (see Appendix E). In most cases, the function is found in a Matlab M-file of the same name in the lib directory of the distribution, where the .m extension is omitted from this listing. For more information on each, at the Matlab prompt, simply type help followed by the name of the function. For documentation and data files, the filename extensions are included.

\section*{D. 1 Directory Layout and Documentation Files}

Some of these files appear at the top level of the distribution and others in the docs directory.

Table D-1: Matpower Directory Layout and Documentation Files
\begin{tabular}{|c|c|}
\hline name & description \\
\hline data/ & Matpower cases (see Section D.3) \\
\hline docs/ & Matpower documentation \\
\hline MATPOWER-dev-guide.md & Matpower Developer's Guide \\
\hline MATPOWER-manual.pdf & Matpower User's Manual \({ }^{\dagger}\) \\
\hline relnotes/ & Matpower release notes \\
\hline src/ & LATEX source for Matpower User's Manual \\
\hline TN1-OPF-Auctions.pdf & Matpower Technical Note 1 "Uniform Price Auctions and Optimal Power Flow" [46] \\
\hline TN2-OPF-Derivatives.pdf & Matpower Technical Note 2 "AC Power Flows, Generalized OPF Costs and their Derivatives using Complex Matrix Notation" [36] \\
\hline TN3-More-OPF-Derivatives.pdf & Matpower Technical Note 3 "Addendum to AC Power Flows and their Derivatives using Complex Matrix Notation: Nodal Current Balance" [37] \\
\hline TN4-OPF-Derivatives-Cartesian.pdf & Matpower Technical Note 4 "AC Power Flows and their Derivatives using Complex Matrix Notation and Cartesian Coordinate Voltages" [38] \\
\hline ```
lib/
    caseformat.m
``` & \begin{tabular}{l}
Matpower software (see Section D.2) \\
help file documenting Matpower case format (i.e. at the command prompt, type help caseformat to see details)
\end{tabular} \\
\hline t/ & Matpower tests and examples (see Section D.4) \\
\hline mips/ & integrated MIPS distribution \({ }^{\dagger}\) \\
\hline most/ & integrated MOST distribution \({ }^{\dagger}\) \\
\hline mptest/ & integrated MP-Test distribution (see Table D-24) \\
\hline AUTHORS & list of Matpower authors \\
\hline CHANGES.md & Matpower change history \\
\hline CONTRIBUTING.md & Matpower Contributors Guide \\
\hline LICENSE & license file \\
\hline README.md & basic introduction to Matpower \\
\hline
\end{tabular}

\footnotetext{
\(\dagger\) The MIPS User's Manual and MOST User's Manual are found in <MATPOWER>/mips/docs and <MATPOWER>/most/docs, respectively.
}

\section*{D. 2 Matpower Functions}

All of the functions listed in this section are found in \(\langle M A T P O W E R>/\) lib, unless noted otherwise.

Table D-2: Top-Level Simulation Functions
\begin{tabular}{ll}
\hline name & description \\
\hline runpf & power flow \(^{\dagger}\) \\
runcpf & AC continuation power flow \\
runopf & optimal power flow \(^{\dagger}\) \\
runuopf & optimal power flow with unit-decommitment \({ }^{\dagger}\) \\
rundcpf & DC power flow \({ }^{\ddagger}\) \\
\begin{tabular}{l} 
rundcopf \\
runduopf \\
runopf_w_res \\
most
\end{tabular} & \begin{tabular}{l} 
DC optimal power flow \\
\\
DC optimal power flow with unit-decommitment \\
\\
optimal power flow with fixed reserve requirements \\
\\
\\
\hline
\end{tabular} \\
\hline
\end{tabular}
\(\dagger\) Uses AC model by default.
\(\ddagger\) Simple wrapper function to set option to use DC model before calling the corresponding general function above.
§ MOST and its supporting files and functions in the most/ sub-directory are documented in the MOST User's Manual and listed in its Appendix A.

Table D-3: Input/Output Functions
\begin{tabular}{ll}
\hline name & description \\
\hline cdf2mpc & \begin{tabular}{l} 
converts power flow data from IEEE Common Data Format (CDF) to \\
MatPowER format
\end{tabular} \\
loadcase & \begin{tabular}{l} 
loads data from a MATPOWER case file or struct into data matrices or a \\
case struct
\end{tabular} \\
mpoption & sets and retrieves MATPOWER options \\
printpf & pretty prints power flow and OPF results \\
psse2mpc & converts power flow data from PSS/E RAW format to MATPOWER format \\
save2psse & \begin{tabular}{l} 
exports MATPOWER case to PSS/E RAW format \\
savecase
\end{tabular} \\
\hline
\end{tabular}

Table D-4: Data Conversion Functions
\begin{tabular}{ll}
\hline name & description \\
\hline ext2int & converts case from external to internal indexing \\
e2i_data & converts arbitrary data from external to internal indexing \\
e2i_field & converts fields in mpc from external to internal indexing \\
int2ext & converts case from internal to external indexing \\
i2e_data & converts arbitrary data from internal to external indexing \\
i2e_field & converts fields in mpc from internal to external indexing \\
get_reorder & returns \(A\) with one of its dimensions indexed \\
set_reorder & assigns \(B\) to \(A\) with one of the dimensions of \(A\) indexed \\
\hline
\end{tabular}

Table D-5: Power Flow Functions
\begin{tabular}{|c|c|}
\hline name & description \\
\hline calc_v_i_sum & implementation of current summation power flow solver \({ }^{\dagger}\) \\
\hline calc_v_pq_sum & implementation of power summation power flow solver \({ }^{\dagger}\) \\
\hline calc_v_y_sum & implementation of admittance summation power flow solver \({ }^{\dagger}\) \\
\hline dcpf & implementation of DC power flow solver \\
\hline fdpf & implementation of fast-decoupled power flow solver \\
\hline gausspf & implementation of Gauss-Seidel power flow solver \\
\hline make_vcorr & calculate voltage corrections for radial power flow \({ }^{\dagger}\) \\
\hline make_zpv & form loop impedance for PV bus handling in radial power flow \({ }^{\dagger}\) \\
\hline newtonpf & implementation of Newton-method power flow solver \\
\hline order_radial & performs oriented ordering of buses and branches for radial power flow \({ }^{\dagger}\) \\
\hline pfsoln & computes branch flows, generator reactive power (and real power for slack bus), updates bus, gen, branch matrices with solved values \\
\hline radial_pf & wrapper for backward/forward sweep power flow solvers \({ }^{\dagger}\) \\
\hline
\end{tabular}
\(\dagger\) For radial networks only.

Table D-6: Continuation Power Flow Functions
\begin{tabular}{|c|c|}
\hline name & description \\
\hline cpf_corrector & computes Newton method corrector steps \\
\hline cpf_current_mpc & construct MATPOWER case struct for current continuation step \\
\hline cpf_default_callback & callback function that accumulates, and optionally plots, results from each iteration \\
\hline cpf_detect_events & detects event intervals and zeros given previous and current event function values \\
\hline cpf_flim_event_cb & callback function to handle 'FLIM' events \\
\hline cpf_flim_event & event function for 'FLIM' events, to detect branch MVA flow limits \\
\hline cpf_nose_event_cb & callback function to handle 'NOSE' events \\
\hline cpf_nose_event & event function for ' NOSE' events, to detect nose point of continuation curve \\
\hline cpf_p_jac & computes partial derivatives of parameterization function \\
\hline cpf_p & computes value of parameterization function at current solution \\
\hline cpf_plim_event_cb & callback function to handle 'PLIM' events \\
\hline cpf_plim_event & event function for 'PLIM' events, to detect generator active power limits \\
\hline cpf_predictor & computes the predicted solution from the current solution and tangent direction \\
\hline cpf_qlim_event_cb & callback function to handle 'QLIM' events \\
\hline cpf_qlim_event & event function for 'QLIM' events, to detect generator reactive power limits \\
\hline cpf_register_callback & registers a CPF callback function to be called by runcpf \\
\hline cpf_register_event & registers a CPF event function to be called by runcpf \\
\hline cpf_tangent & computes the normalized tangent vector at the current solution \\
\hline cpf_target_lam_event_cb & callback function to handle 'TARGET_LAM' events \\
\hline cpf_target_lam_event & event function for 'TARGET_LAM' events, to detect end of continuation curve or other target \(\lambda\) value \\
\hline cpf_vlim_event_cb & callback function to handle 'VLIM' events \\
\hline cpf_vlim_event & event function for 'VLIM' events, to detect bus voltage magnitude limits \\
\hline
\end{tabular}

Table D-7: OPF and Wrapper Functions
\begin{tabular}{ll}
\hline name & description \\
\hline opf \(^{\dagger}\) & the main OPF function, called by runopf \\
dcopf \(^{\ddagger}\) & calls opf with options set to solve DC OPF \\
fmincopf \(^{\ddagger}\) & calls opf with options set to use fmincon to solve AC OPF \\
mopf \(^{\ddagger}\) & calls opf with options set to use MINOPF to solve AC OPF \\
uopf \(^{\ddagger}\) & implements unit-decommitment heuristic, called by runuopf \\
\hline
\end{tabular}
\(\dagger\) Can also be used as a top-level function, run directly from the command line. It provides more calling options than runopf, primarly for backward compatibility with previous versions of mopf from MINOPF, but does not offer the option to save the output or the solved case.
\(\ddagger\) Wrapper with same calling conventions as opf.
\(\S\) Requires the installation of an optional package. See Appendix G for details on the corresponding package.

Table D-8: OPF Model Objects
\begin{tabular}{ll}
\hline name & description \\
\hline @opf_model/ & OPF model object used to encapsulate OPF problem formulation \\
display & called to display object when statement not ended with semicolon \\
get_mpc & returns the MATPowER case struct \\
opf_model & constructor for the opf_model class \\
@opt_model/ & optimization model object (@opf model base class) \\
add_legacy_cost & adds a named subset of legacy user-defined costs to the model \\
add_lin_constraint & adds a named subset of linear constraints to the model \\
add_named_set & adds a named subset of costs, constraints or variables to the model \\
add_nln_constraint & adds a named subset of nonlinear constraints to the model \\
add_nln_cost & adds a named subset of general nonlinear costs to the model \\
add_quad_cost & adds a named subset of quadratic costs to the model \\
add_var & adds a named subset of optimization variables to the model \\
describe_idx & identifies variable, constraint or cost row indices \\
display & called to display object when statement not ended with semicolon \\
eval_legacy_cost & evaluates legacy user costs and derivatives \\
eval_nln_constraint & returns full set of nonlinear equality or inequality constraints and \\
eval_nln_constraint_hess & their gradients \\
& returns Hessian for full set of nonlinear equality or inequality con- \\
eval_nln_cost & straints \\
eval_quad_cost & evaluates general nonlinear costs and derivatives \\
get_idx & evaluates quadratic costs and derivatives \\
get_userdata & returns the idx struct for vars, lin/nln constraints, costs \\
getN & returns values of user data stored in the model \\
get & returns the number of variables, constraints or cost rows \\
init_indexed_name & returns the value of a field of the object \\
opt_model & initializes dimensions for indexed named set of costs, constraints \\
params_lin_constraint & or variables \\
params_legacy_cost & constructor for the opt_model class \\
params_nln_cost & returns individual or full set of linear constraint parameters set list \\
params_quad_cost & returns individual or full set of legacy user cost coefficients \\
params_var & returns individual general nonlinear cost parameters \\
returns individual or full set of quadratic cost coefficients
\end{tabular}

\footnotetext{
\(\dagger\) Private method for internal use only.
\(\ddagger\) For all, or alternatively, only for a named (and possibly indexed) subset.
}

Table D-9: Deprecated @opt_model Methods
\begin{tabular}{ll}
\hline name & description \\
\hline @opt_model/ & optimization model object (@opf_model base class) \\
add_constraints & use add_lin_constraint or add_nln_constraint instead \({ }^{\dagger}\) \\
add_costs \(^{\text {add_vars }}\) & use add_legacy_cost, add_nln_cost or add_quad_cost instead \\
\\
build_cost_params \(^{\dagger}\) & use add_var instead \\
compute_cost & no longer needed \\
get_cost_params & use eval_legacy_cost instead \\
getv & use params_legacy_cost instead \\
linear_constraints & use params_var instead \\
\hline
\end{tabular}
\(\dagger\) Or init_indexed_name if simply initializing the dimensions for an indexed named set.

Table D-10: OPF Solver Functions
\begin{tabular}{ll}
\hline name & description \\
\hline dcopf_solver & sets up and solves DC OPF problem \\
fmincopf_solver & sets up and solves OPF problem using fmincon, MATLAB Opt Toolbox \\
ipoptopf_solver & sets up and solves OPF problem using IPOPT \({ }^{\dagger}\) \\
ktropf_solver & sets up and solves OPF problem using KNITRO \({ }^{\dagger}\) \\
mipsopf_solver & sets up and solves OPF problem using MIPS \\
mopf_solver & sets up and solves OPF problem using MINOPF \({ }^{\dagger}\) \\
tspopf_solver & sets up and solves OPF problem using PDIPM, SC-PDIPM or TRALM \({ }^{\dagger}\) \\
\hline\(\dagger\) Requires the installation of an optional package. See Appendix G for details on the corresponding package.
\end{tabular}

Table D-11: Other OPF Functions
\begin{tabular}{|c|c|}
\hline name & description \\
\hline margcost & computes the marginal cost of generation as a function of generator output \\
\hline makeAang & forms linear constraints for branch angle difference limits \\
\hline makeApq & forms linear constraints for generator PQ capability curves \\
\hline makeAvl & forms linear constraints for dispatchable load constant power factor \\
\hline makeAy & forms linear constraints for piecewise linear generator costs (CCV) \\
\hline opf_args & input argument handling for opf \\
\hline opf_setup & constructs an OPF model object from a Matpower case \\
\hline opf_execute & executes the OPF specified by an OPF model object \\
\hline opf_branch_ang_fen \({ }^{\dagger}\) & evaluates AC branch angle difference limit constraints and gradients \({ }^{\ddagger}\) \\
\hline opf_branch_ang_hess \({ }^{\dagger}\) & evaluates Hessian of AC branch angle difference limit constraints \({ }^{\ddagger}\) \\
\hline opf_branch_flow_f cn \({ }^{\dagger}\) & evaluates AC branch flow limit constraints and gradients \\
\hline opf_branch_flow_hess \({ }^{\dagger}\) & evaluates Hessian of AC branch flow limit constraints \\
\hline opf_consfen \({ }^{\dagger}\) & evaluates function and gradients for AC OPF nonlinear constraints \\
\hline opf_costfen \({ }^{\dagger}\) & evaluates function, gradients and Hessian for AC OPF objective function \\
\hline opf_current_balance_fen \({ }^{\dagger}\) & evaluates AC current balance constraints and gradients \\
\hline opf_current_balance_hess \({ }^{\dagger}\) & evaluates Hessian of AC current balance constraints \\
\hline opf_gen_cost_f cn \({ }^{\dagger}\) & evaluates polynomial generator costs and derivatives \\
\hline opf_hessfcn \({ }^{\dagger}\) & evaluates the Hessian of the Lagrangian for AC OPF \\
\hline opf_legacy_user_cost_fen \({ }^{\dagger}\) & evaluates legacy user-defined costs and derivatives \\
\hline opf_power_balance_fen \({ }^{\dagger}\) & evaluates AC power balance constraints and gradients \\
\hline opf_power_balance_hess \({ }^{\dagger}\) & evaluates Hessian of AC power balance constraints \\
\hline opf_veq_fcn \({ }^{\dagger}\) & evaluates voltage magnitude equality constraints and gradients \({ }^{\ddagger}\) \\
\hline opf_veq_hess \({ }^{\dagger}\) & evaluates Hessian of voltage magnitude equality constraints \({ }^{\ddagger}\) \\
\hline opf_vlim_fen \({ }^{\dagger}\) & evaluates voltage magnitude limit constraints and gradients \({ }^{\ddagger}\) \\
\hline opf_vlim_hess \({ }^{\dagger}\) & evaluates Hessian of voltage magnitude limit constraints \({ }^{\ddagger}\) \\
\hline opf_vref_f \(\mathrm{cn}^{\dagger}\) & evaluates reference voltage angle equality constraints and gradients \({ }^{\ddagger}\) \\
\hline opf_vref_hess \({ }^{\dagger}\) & evaluates Hessian of reference voltage angle equality constraints \({ }^{\ddagger}\) \\
\hline totcost & computes the total cost of generation as a function of generator output \\
\hline update_mupq & updates generator limit prices based on the shadow prices on capability curve constraints \\
\hline
\end{tabular}

\footnotetext{
\(\dagger\) Used by fmincon, MIPS, Ipopt and KNITRO for AC OPF.
\(\ddagger\) Used with cartesian coordinate voltage representation only. I.e. opf.v_cartesian option set to 1.
}

Table D-12: OPF User Callback Functions
\begin{tabular}{ll}
\hline name & description \\
\hline add_userfcn & \begin{tabular}{l} 
appends a user callback function to the list of those to be called for a \\
given case \\
removes a user callback function from the list \\
run_userfcn \\
toggle_dcline
\end{tabular} \\
\begin{tabular}{l} 
executes the user callback functions for a given stage \\
enable/disable or check the status of the callbacks implementing DC \\
lines \\
enable/disable or check the status of the callbacks implementing in- \\
toggle_iflims \\
toggle_reserves \\
enable/disable or check the status of the callbacks implementing fixed \\
reserve requirements \\
enable/disable or check the status of the callbacks implementing DC \\
OPF branch flow soft limits
\end{tabular} \\
\hline
\end{tabular}

Table D-13: Power Flow Derivative Functions
\begin{tabular}{|c|c|}
\hline name & description \({ }^{\dagger}\) \\
\hline dIbr_dV & evaluates the partial derivatives of \(I_{f \mid t}\) with respect \\
\hline dSbr_dV & evaluates the partial derivatives of \(S_{f \mid t}\) with respect to \(V\) \\
\hline dAbr_dV & evaluates the partial derivatives of \(\left|F_{f \mid t}\right|^{2}\) with respect to \(V\) \\
\hline dImis_dV & evaluates the partial derivatives of \(I_{\text {mis }}\) with respect to \(V\) \\
\hline dSbus_dV & evaluates the partial derivatives of \(S_{\text {bus }}\) with respect to \(V\) \\
\hline d2Ibr_dV2 & evaluates the \(2^{\text {nd }}\) derivatives of \(I_{f \mid t}\) with respect to \(V\) \\
\hline d2Sbr_dV2 & evaluates the \(2^{\text {nd }}\) derivatives of \(S_{f \mid t}\) with respect to \(V\) \\
\hline d2Abr_dV2 & evaluates the \(2^{\text {nd }}\) derivatives of \(\left|F_{f \mid t}\right|^{2}\) with respect to \(V\) \\
\hline d2AIbr_dV2* & evaluates the \(2^{\text {nd }}\) derivatives of \(\left|I_{f \mid t}\right|^{2}\) with respect to \(V\) \\
\hline d2ASbr_dV2* & evaluates the \(2^{\text {nd }}\) derivatives of \(\left|S_{f \mid t}\right|^{2}\) with respect to \(V\) \\
\hline d2Imis_dV2 & evaluates the \(2^{\text {nd }}\) derivatives of \(I_{\text {mis }}\) with respect to \(V\) \\
\hline d2Imis_dVdSg & evaluates the \(2^{\text {nd }}\) derivatives of \(I_{\text {mis }}\) with respect to \(V\) and \(S_{g}\). \\
\hline d2Sbus_dV2 & evaluates the \(2^{\text {nd }}\) derivatives of \(S_{\text {bus }}\) with respect to \(V\) \\
\hline \multicolumn{2}{|l|}{\multirow[t]{2}{*}{\begin{tabular}{l}
* Deprecated. Please use d2Abr_dV2 instead. \\
\(\dagger V\) represents complex bus voltages, \(S_{g}\) the complex generator power injections, \(I_{f \mid t}\) complex branch current injections, \(S_{f \mid t}\) complex branch power injections, \(I_{\text {mis }}\) complex bus current injections, \(S_{\text {bus }}\) complex bus power injections and \(F_{f \mid t}\) refers to branch flows, either \(I_{f \mid t}\) or \(S_{f \mid t}\), depending on the inputs. The second derivatives are all actually partial derivatives of the product of a first derivative matrix and a vector \(\lambda\). Please see Matpower Technical Note 2 [36], Matpower Technical Note 3 [37] and Matpower Technical Note 4 [38] for the detailed derivations of the formulas implemented by these functions.
\end{tabular}}} \\
\hline & \\
\hline
\end{tabular}

Table D-14: NLP, LP \& QP Solver Functions
\begin{tabular}{|c|c|}
\hline name & description \\
\hline clp_options & default options for CLP solver \({ }^{\dagger}\) \\
\hline cplex_options & default options for CPLEX solver \({ }^{\dagger}\) \\
\hline glpk_options & default options for GLPK solver \({ }^{\dagger}\) \\
\hline gurobi_options & default options for Gurobi solver \({ }^{\dagger}\) \\
\hline gurobiver & prints version information for Gurobi/Gurobi_MEX \\
\hline ipopt_options & default options for IPOPT solver \({ }^{\dagger}\) \\
\hline \begin{tabular}{l}
mips \\
mipsver
\end{tabular} & Matpower Interior Point Solver - primal/dual interior point solver for NLP prints version information for MIPS \\
\hline miqps matpower & Mixed-Integer Quadratic Program Solver for Matpower, wrapper function provides a common MIQP solver interface for various MIQP/MILP solvers \\
\hline miqps_cplex & common MIQP/MILP solver interface to CPLEX (cplexmiqp and cplexmilp \()^{\dagger}\) \\
\hline miqps_glpk & common MILP solver interface to GLPK \({ }^{\dagger}\) \\
\hline miqps_gurobi & common MIQP/MILP solver interface to Gurobi \({ }^{\dagger}\) \\
\hline miqps_mosek & common MIQP/MILP solver interface to MOSEK (mosekopt) \({ }^{\dagger}\) \\
\hline miqps_ot & common QP/MILP solver interface to MatLab Opt Toolbox's intlinprog, quadprog, linprog \\
\hline mosek_options & default options for MOSEK solver \({ }^{\dagger}\) \\
\hline mosek_symbcon & symbolic constants to use for MOSEK solver options \({ }^{\dagger}\) \\
\hline mplinsolve & common linear system solver interface, used by MIPS \\
\hline mpopt2qpopt & create mi/qps_matpower options struct from Matpower options struct \\
\hline qps_matpower & Quadratic Program Solver for MatPower, wrapper function provides a common QP solver interface for various QP/LP solvers \\
\hline qps_bpmpd & common QP/LP solver interface to BPMPD_MEX \({ }^{\dagger}\) \\
\hline qps_clp & common QP/LP solver interface to \(\mathrm{CLP}^{\dagger}\) \\
\hline qps_cplex & common QP/LP solver interface to CPLEX (cplexqp and cplexlp) \({ }^{\dagger}\) \\
\hline qps_glpk & common QP/LP solver interface to GLPK \({ }^{\dagger}\) \\
\hline qps_gurobi & common QP/LP solver interface to Gurobi \({ }^{\dagger}\) \\
\hline qps_ipopt & common QP/LP solver interface to IPOPT-based solver \({ }^{\dagger}\) \\
\hline qps_mips & common QP/LP solver interface to MIPS-based solver \\
\hline qps_mosek & common QP/LP solver interface to MOSEK (mosekopt) \({ }^{\dagger}\) \\
\hline qps_ot & common QP/LP solver interface to Matlab Opt Toolbox's quadprog, linprog \\
\hline
\end{tabular}

\footnotetext{
\(\dagger\) Requires the installation of an optional package. See Appendix G for details on the corresponding package.
}

Table D-15: Matrix Building Functions
\(\left.\begin{array}{ll}\hline \text { name } & \text { description } \\
\hline \text { makeB } & \text { forms the fast-decoupled power flow matrices, } B^{\prime} \text { and } B^{\prime \prime} \\
\text { makeBdc } & \text { forms the system matrices } B_{\text {bus }} \text { and } B_{f} \text { and vectors } P_{f, \text { shift }} \text { and } P_{\text {bus,shift }} \\
\text { for the DC power flow model }\end{array}\right]\)\begin{tabular}{ll} 
makeJac & forms the power flow Jacobian matrix \\
makeLODF & forms the line outage distribution factor matrix \\
makePTDF & forms the DC PTDF matrix for a given choice of slack \\
makeSbus & forms the vector of complex bus power injections \\
makeSdzip & forms a struct w/vectors of nominal values for each component of ZIP load \\
makeYbus & forms the complex bus and branch admittance matrices \(Y_{\text {bus }}, Y_{f}\) and \(Y_{t}\) \\
\hline
\end{tabular}

Table D-16: Utility Functions
\begin{tabular}{ll}
\hline name & description \\
\hline apply_changes & \begin{tabular}{l} 
modifies an existing MATPOWER case by applying changes defined \\
in a change table
\end{tabular} \\
bustypes & creates vectors of bus indices for reference bus, PV buses, PQ buses \\
calc_branch_angle & calcultes a vector of voltage angle differences for branches \\
case_info & checks a MATPOWER case for connectivity and prints a system \\
& summary \\
compare_case & prints summary of differences between two MATPowER cases \\
define_constants & convenience script defines constants for named column indices to \\
& data matrices (calls idx_bus, idx_brch, idx_gen and idx_cost) \\
extract_islands & extracts islands in a network into their own MATPowER case struct \\
feval_w_path & same as MATLAB's feval function, except the function being eval- \\
& uated need not be in the MATLAB path \\
find_islands & finds islands and isolated buses in a network \\
genfuels & list of standard values for generator fuel types, for mpc.genfuel \\
gentypes & list of standard values for generator unit types, for mpc.gentype \\
get_losses & compute branch losses (and derivatives) as functions of bus voltage \\
hasPQcap & checks for generator P-Q capability curve constraints \\
have_fcn & checks for availability of optional functionality \\
idx_brch & named column index definitions for branch matrix \\
idx_bus & named column index definitions for bus matrix \\
idx_cost & named column index definitions for gencost matrix \\
idx_ct & constant definitions for use with change tables and apply_changes \\
idx_dcline & named column index definitions for dcline matrix \\
idx_gen & named column index definitions for gen matrix \\
isload & checks if generators are actually dispatchable loads \\
load2disp & converts fixed loads to dispatchable loads \\
loadshed & computes MW curtailments of dispatchable loads \\
mpver & prints version information for MatPowER and optional packages \\
modcost & modifies gencost by horizontal or vertical scaling or shifting \\
nested_struct_copy & copies the contents of nested structs \\
poly2pwl & creates piecewise linear approximation to polynomial cost function \\
polycost & evaluates polynomial generator cost and its derivatives \\
pqcost & splits gencost into real and reactive power costs \\
savechgtab & saves a change table, such as used by apply_changes, to a file \\
scale_load & scales fixed and/or dispatchable loads by load zone \\
total_load & returns vector of total load in each load zone \\
\hline
\end{tabular}

Table D-17: Other Functions
\begin{tabular}{ll}
\hline name & description \\
\hline connected_components & returns the connected components of a graph \\
mpoption_info_clp & option information for CLP \\
mpoption_info_cplex & option information for CPLEX \\
mpoption_info_fmincon & option information for FMINCON \\
mpoption_info_glpk & option information for GLPK \\
mpoption_info_gurobi & option information for Gurobi \\
mpoption_info_intlinprog & option information for INTLINPROG \\
mpoption_info_ipopt & option information for IPOPT \\
mpoption_info_knitro & option information for KNITRO \\
mpoption_info_linprog & option information for LINPROG \\
mpoption_info_mips & option information for MIPS \\
mpoption_info_mosek & option information for MOSEK \\
mpoption_info_quadprog & option information for QUADPROG \\
psse_convert & converts data read from PSS/E RAW file to MATPOWER format \\
psse_convert_hvdc & called by psse_convert to handle HVDC data \\
psse_convert_xfmr & called by psse_convert to handle transformer data \\
psse_parse & parses data from a PSS/E RAW data file \\
psse_parse_line & called by psse_parse to parse a single line \\
psse_parse_section & called by psse_parse to parse an entire section \\
psse_read & reads data from a PSS/E RAW data file \\
\hline
\end{tabular}

\section*{D. 3 Example Matpower Cases}

All of the functions listed in this section are found in \(<M A T P O W E R>/\) data, unless noted otherwise.

Table D-18: Small Transmission System Test Cases
\begin{tabular}{ll}
\hline name & description \\
\hline case4gs & \begin{tabular}{l} 
4-bus example case from Grainger \& Stevenson \\
case5
\end{tabular} \\
case6ww & modified 5-bus PJM example case from Rui Bo \\
case9 & 6-bus example case from Wood \& Wollenberg \\
case9Q & 9-bus example case from Chow \\
case9target & case9 with reactive power costs \\
case14 & IEEE 14-bus case \\
case24_ieee_rts & IEEE RTS 24-bus case \\
case30 & 30-bus case, based on IEEE 30-bus case \\
case_ieee30 & IEEE 30-bus case \\
case30pwl & case30 with piecewise linear costs \\
case30Q & case30 with reactive power costs \\
case39 & 39-bus New England case \\
case57 & IEEE 57-bus case \\
case_RTS_GMLC & 96-machine, 73-bus Reliability Test System* \\
case118 & IEEE 118-bus case \\
case145 & IEEE 145-bus case, 50 generator dynamic test case \\
case300 & IEEE 300-bus case \\
\hline
\end{tabular}
* Reliability Test System Grid Modernization Lab Consortium data from https://github.com/GridMod/RTS-GMLC.

Table D-19: Small Radial Distribution System Test Cases
\begin{tabular}{ll}
\hline name & description \\
\hline case4_dist & 4-bus example radial distribution system case \\
case18 & 18-bus radial distribution system from Grady, Samotyj and Noyola \\
case22 & 22-bus radial distribution system from Raju, Murthy and Ravindra \\
case33bw & 33-bus radial distribution system from Baran and Wu \\
case69 & 69-bus radial distribution system from Das \\
case85 & 85-bus radial distribution system from Das, Kothari and Kalam \\
case141 & 141-bus radial distribution system from Khodr, Olsina, De Jesus and Yusta \\
\hline
\end{tabular}

Table D-20: ACTIV Synthetic Grid Test Cases
\begin{tabular}{|c|c|}
\hline name & description \\
\hline case_ACTIVSg200 & 200-bus Illinois synthetic model \({ }^{*}\) \\
\hline case_ACTIVSg500 & 500-bus South Carolina synthetic model* \\
\hline case_ACTIVSg2000 & 2000-bus Texas synthetic model \({ }^{*}\) \\
\hline case_ACTIVSg10k & 10,000-bus US WECC synthetic model \({ }^{*}\) \\
\hline case_ACTIVSg25k & 25,000-bus US Northeast/Mid-Atlantic synthetic model \({ }^{*}\) \\
\hline case_ACTIVSg70k & 70,000-bus Eastern US synthetic model \({ }^{*}\) \\
\hline case_SyntheticUSA & 82,000-bus continental USA synthetic model* \\
\hline contab_ACTIVSg200 & contingency table for case_ACTIVSg200* \\
\hline contab_ACTIVSg500 & contingency table for case_ACTIVSg500* \\
\hline contab_ACTIVSg2000 & contingency table for case_ACTIVSg2000* \\
\hline contab_ACTIVSg10k & contingency table for case_ACTIVSg10k* \\
\hline scenarios_ACTIVSg200 & one year of hourly zonal load scenarios for case_ACTIVSg200* \\
\hline scenarios_ACTIVSg2000 & one year of hourly area load scenarios for case_ACTIVSg2000* \\
\hline
\end{tabular}
* These synthetic grid models are fictitious representations that are designed to be statistically and functionally similar to actual electric grids while containing no confidential critical energy infrastructure information (CEII). Please cite reference [42] when publishing results based on this data. More information about these synthetic cases can be found at https://electricgrids.engr.tamu.edu.

Table D-21: Polish System Test Cases
\begin{tabular}{ll}
\hline name & description \\
\hline case2383wp & Polish system - winter 1999-2000 peak \\
case2736sp & Polish system - summer 2004 peak
\end{tabular}
* Contributed by Roman Korab.

Table D-22: PEGASE European System Test Cases
\begin{tabular}{ll}
\hline name & description \\
\hline case89pegase & 89-bus portion of European transmission system from PEGASE project
\end{tabular}

Table D-23: RTE French System Test Cases
\begin{tabular}{ll}
\hline name & description \\
\hline case1888rte & 1888-bus snapshot of VHV French transmission system from RTE \({ }^{*}\) \\
case1951rte & 1951-bus snapshot of VHV French transmission system from RTE \\
case2848rte & 2848-bus snapshot of VHV French transmission system from RTE \({ }^{*}\) \\
case2868rte & 2868-bus snapshot of VHV French transmission system from RTE \\
case6468rte & 6468-bus snapshot of VHV and HV French transmission system from RTE \\
case6470rte & 6470-bus snapshot of VHV and HV French transmission system from RTE \\
case6495rte & 6495-bus snapshot of VHV and HV French transmission system from RTE \\
case6515rte & 6515-bus snapshot of VHV and HV French transmission system from RTE
\end{tabular}

\footnotetext{
* Contributed by Cédric Josz and colleagues from the French Transmission System Operator. Please cite reference [43] when publishing results based on this data.
}

\section*{D. 4 Automated Test Suite}

All of the functions and files listed in this section are found in \(\langle M A T P O W E R>/ l i b / t\), unless noted otherwise.

Table D-24: Automated Test Functions from MP-Test \({ }^{\dagger}\)
\begin{tabular}{ll}
\hline name & description \\
\hline mptest/lib/ & \\
t_begin & begin running tests \\
t_end & finish running tests and print statistics \\
t_is & tests if two matrices are identical to with a specified tolerance \\
t_ok & tests if a condition is true \\
t_run_tests & run a series of tests \\
t_skip & skips a number of tests, with explanatory message \\
t/ & MP-Test tests \\
test_mptest & runs full MP-Test test suite \\
t_test_fcns & test t_ok and t_is \\
\hline
\end{tabular}
\(\dagger\) These functions are part of MP-Test and are found in <MATPOWER \(/\) /mptest/lib.

Table D-25: MIPS Tests \({ }^{\dagger}\)
\begin{tabular}{ll}
\hline name & description \\
\hline mips/lib/ & \\
t/ & MIPS tests \\
test_mips & runs full MIPS test suite \\
t_mips & runs tests for MIPS NLP solver \\
t_mips_pardiso & runs tests for MIPS NLP solver, using PARDISO as linear solver \\
t_mplinsolve & tests for mplinsolve \\
t_qps_mips & runs tests for qps_mips \\
\hline
\end{tabular}
\(\dagger\) These tests are part of MIPS and are found in \(\langle M A T P O W E R>/ \mathrm{mips} / \mathrm{lib} / \mathrm{t}\).

Table D-26: Test Data
\begin{tabular}{ll}
\hline name & description \\
\hline opf_nle_fcn1.m & user-defined nonlinear equality constraint function for OPF tests \\
opf_nle_hess1.m & user-defined nonlinear equality constraint Hessian for OPF tests \\
pretty_print_acopf.txt & pretty-printed output of an example AC OPF run \\
pretty_print_dcopf.txt & pretty-printed output of an example DC OPF run \\
soln9_dcopf.mat & solution data, DC OPF of t_case9_opf \\
soln9_dcpf.mat & solution data, DC power flow of t_case9_pf \\
soln9_opf_ang.mat & solution data, AC OPF of t_case9_opfv2 w/only branch angle \\
& difference limits (gen PQ capability constraints removed) \\
soln9_opf_extras1.mat & solution data, AC OPF of t_case9_opf w/extra cost/constraints \\
soln9_opf_Plim.mat & solution data, AC OPF of t_case9_opf w/opf.flow_lim \(=\) 'P' \\
soln9_opf_PQcap.mat & solution data, AC OPF of t_case9_opfv2 w/only gen PQ capabil- \\
& ity constraints (branch angle diff limits removed) \\
soln9_opf_vg.mat & solution data, AC OPF of t_case9_opf with option opf.use_vg \\
soln9_opf.mat & solution data, AC OPF of t_case9_opf \\
soln9_pf.mat & solution data, AC power flow of t_case9_pf \\
t_auction_case.m & case data used to test auction code \\
t_case_ext.m & case data used to test ext2int and int2ext, external indexing \\
t_case_info_eg.txt & example output from case_info, used by t_islands \\
t_case_int.m & case data used to test ext2int and int2ext, internal indexing \\
t_case9_dcline.m & same as t_case9_opfv2 with additional DC line data \\
t_case9_opf.m & sample case file with OPF data, version 1 format \\
t_case9_opfv2.m & sample case file with OPF data, version 2 format, includes addi- \\
t_case9_pf.m & tional branch angle diff limits \& gen PQ capability constraints \\
t_case9_pfv2.m & sample case file with only power flow data, version 1 format \\
t_case9_save2psse.m & sample case file with only power flow data, version 2 format \\
t_case30_userfcns.m & sample 9-bus case file used by t_psse for testing save2psse \\
t_chgtab.m & sample case file with OPF, reserves and interface flow limit data \\
t_psse_case.raw & sample change table data, used by t_apply_changes \\
t_psse_case2.m & sample PSS/E format RAW file used to test conversion code \\
t_psse_case2.raw & result of converting t_psse_case2.raw to MatPowER format \\
t_psse_case3.m & sample PSS/E format RAW file used to test conversion code \\
t_psse_case3.raw & result of converting t_psse_case3.raw to MaTPowER format \\
\hline & sample PSS/E format RAW file used to test conversion code \\
\hline
\end{tabular}

Table D-27: Miscellaneous Matpower Tests
\begin{tabular}{ll}
\hline name & description \\
\hline test_matpower & runs full MATPOWER test suite \\
test_most \(\ddagger\) & runs full MOST test suite \\
t_apply_changes & runs tests for apply_changes \\
t_auction_minopf & runs tests for auction using MINOPF \({ }^{\dagger}\) \\
t_auction_mips & runs tests for auction using MIPS \\
t_auction_tspopf_pdipm & runs tests for auction using PDIPM \({ }^{\dagger}\) \\
t_cpf & runs tests for AC continuation power flow \\
t_cpf_cb1 & example CPF callback function for t_cpf \\
t_cpf_cb2 & example CPF callback function with cb_args for t_cpf \\
t_dcline & runs tests for DC line implementation in toggle_dcline \\
t_ext2int2ext & runs tests for ext2int and int2ext \\
t_get_losses & runs tests for get_losses \\
t_hasPQcap & runs tests for hasPQcap \\
t_hessian & runs tests for \(2^{\text {nd }}\) derivative code \\
t_islands & runs test for find_islands and extract_islands \\
t_jacobian & runs test for partial derivative code \\
t_load2disp & runs tests for load2disp \\
t_loadcase & runs tests for loadcase \\
t_makeLODF & runs tests for makeLODF \\
t_makePTDF & runs tests for makePTDF \\
t_margcost & runs tests for margcost \\
t_miqps_matpower & runs tests for miqps_matpower \\
t_modcost & runs tests for modcost \\
t_mpoption & runs tests for mpoption \\
t_nested_struct_copy & runs tests for nested_struct_copy \\
t_off2case & runs tests for off2case \\
t_opf_model & runs tests for opf_model and opt_model objects \\
t_opf_model_legacy & runs tests for opf_model and opt_model objects using legacy (dep- \\
t_printpf & recated) add_constraints method \\
t_psse & runs tests for printpf \\
t_qps_matpower & runs tests for psse2mpc and related functions \\
t_pf & runs tests for qps_matpower \\
t_pf_radial & runs tests for AC and DC power flow \\
t_runmarket & runs tests for AC power flow for radial distribution systems \\
t_scale_load & runs tests for runmarket \\
t_total_load & runs tests for scale_load \\
t_vdep_load & runs tests for total_load \\
\hline & runs
\end{tabular}

\footnotetext{
\(\dagger\) Requires the installation of an optional package. See Appendix G for details on the corresponding package.
\(\ddagger\) While test most is listed here with other tests, it is actually located in \(\langle M A T P O W E R>/ \mathrm{most} / \mathrm{lib} / \mathrm{t}\), not <MATPOWER \(/\) /lib/t. MOST and its supporting files and functions in the most/ sub-directory are documented in the MOST User's Manual and listed in its Appendix A.
}

Table D-28: Matpower OPF Tests
\begin{tabular}{|c|c|}
\hline name & description \\
\hline t_opf_dc_bpmpd & runs tests for DC OPF solver using BPMPD_MEX \({ }^{\dagger}\) \\
\hline t_opf_dc_clp & runs tests for DC OPF solver using CLP \({ }^{\dagger}\) \\
\hline t_opf_dc_cplex & runs tests for DC OPF solver using CPLEX \({ }^{\dagger}\) \\
\hline t_opf_dc_glpk & runs tests for DC OPF solver using GLPK \({ }^{\dagger}\) \\
\hline t_opf_dc_gurobi & runs tests for DC OPF solver using Gurobi \({ }^{\dagger}\) \\
\hline t_opf_dc_ipopt & runs tests for DC OPF solver using IPOPT \({ }^{\dagger}\) \\
\hline t_opf_dc_mosek & runs tests for DC OPF solver using MOSEK \({ }^{\dagger}\) \\
\hline t_opf_dc_ot & runs tests for DC OPF solver using Matlab Opt Toolbox \\
\hline t_opf_dc_mips & runs tests for DC OPF solver using MIPS \\
\hline t_opf_dc_mips_sc & runs tests for DC OPF solver using MIPS-sc \\
\hline t_opf_default & runs tests for AC OPF solver using default solver \\
\hline t_opf_fmincon & runs tests for AC OPF solver using fmincon \\
\hline t_opf_ipopt & runs tests for AC OPF solver using IPOPT \({ }^{\dagger}\) \\
\hline t_opf_knitro & runs tests for AC OPF solver using KNITRO \({ }^{\dagger}\) \\
\hline t_opf_minopf & runs tests for AC OPF solver using MINOPF \({ }^{\dagger}\) \\
\hline t_opf_mips & runs tests for AC OPF solver using MIPS \\
\hline t_opf_mips_sc & runs tests for AC OPF solver using MIPS-sc \\
\hline t_opf_softlims & runs tests for DC OPF with user callback functions for branch flow soft limits \\
\hline t_opf_tspopf_pdipm & runs tests for AC OPF solver using \(\mathrm{PDIPM}^{\dagger}\) \\
\hline t_opf_tspopf_scpdipm & runs tests for AC OPF solver using SC-PDIPM \({ }^{\dagger}\) \\
\hline t_opf_tspopf_tralm & runs tests for AC OPF solver using TRALM \({ }^{\dagger}\) \\
\hline t_opf_userfens & runs tests for AC OPF with user callback functions for reserves and interface flow limits \\
\hline t_runopf_w_res & runs tests for AC OPF with fixed reserve requirements \\
\hline
\end{tabular}

\footnotetext{
\(\dagger\) Requires the installation of an optional package. See Appendix G for details on the corresponding package.
}

\section*{Appendix E Extras Directory}

For a MATPOWER installation in \(<\) MATPOWER \(>\), the contents of \(\langle M A T P O W E R>\) /extras contains additional Matpower related code, some contributed by others. Some of these could be moved into the main Matpower distribution in the future with a bit of polishing and additional documentation. Please contact the developers if you are interested in helping make this happen.
maxloadlim An optimal power flow extension, contributed by Camille Hamon, for computing maximum loadability limits. \({ }^{48}\) Please see extras/maxloadlim/manual/maxloadlim_manual.pdf for details on the formulation and implementation.
misc A number of potentially useful functions that are either not yet fully implemented, tested, documented and/or supported. See the help (and the code) in each individual file to understand what it does. \({ }^{49}\)
reduction A network reduction toolbox that performs a modifed Ward reduction and can be used to produce a smaller approximate equivalent from a larger original system. Code contributed by Yujia Zhu and Daniel Tylavsky. For more details, please see the Network_Reduction_Toolbox.pdf file.
sdp_pf Applications of a semidefinite programming programming relaxation of the power flow equations. Code contributed by Dan Molzahn. See Appendix G. 12 and the documentation in the \(<M A T P O W E R>/\) extras/sdp_pf/documentation directory, especially the file sdp_pf_documentation.pdf, for a full description of the functions in this package.
se State-estimation code contributed by Rui Bo. Type test_se, test_se_14bus or test_se_14bus_err to run some examples. See se_intro.pdf for a brief introduction to this code.
smartmarket Code that implements a "smart market" auction clearing mechanism based on Matpower's optimal power flow solver. See Appendix F for details.

\footnotetext{
\({ }^{48}\) Camille Hamon maintains a GitHub repository of this code at: https://github.com/CamilleH/Max-Load-Lim-matpower
\({ }^{49}\) For more information on qcqp_opf, see [43]. For more information on plot_mpc, see [45].
}
state_estimator Older state estimation example, based on code by James S. Thorp.

\section*{Appendix F "Smart Market" Code}

MATPOWER 3 and later includes in the extras/smartmarket directory code that implements a "smart market" auction clearing mechanism. The purpose of this code is to take a set of offers to sell and bids to buy and use Matpower's optimal power flow to compute the corresponding allocations and prices. It has been used extensively by the authors with the optional MINOPF package [30] in the context of PowerWeb \({ }^{50}\) but has not been widely tested in other contexts.

The smart market algorithm consists of the following basic steps:
1. Convert block offers and bids into corresponding generator capacities and costs.
2. Run an optimal power flow with decommitment option (uopf) to find generator allocations and nodal prices \(\left(\lambda_{P}\right)\).
3. Convert generator allocations and nodal prices into set of cleared offers and bids.
4. Print results.

For step 1, the offers and bids are supplied as two structs, offers and bids, each with fields \(P\) for real power and \(Q\) for reactive power (optional). Each of these is also a struct with matrix fields qty and pre, where the element in the \(i\)-th row and \(j\)-th column of qty and prc are the quantity and price, respectively of the \(j\)-th block of capacity being offered/bid by the \(i\)-th generator. These block offers/bids are converted to the equivalent piecewise linear generator costs and generator capacity limits by the off2case function. See help off2case for more information.

Offer blocks must be in non-decreasing order of price and the offer must correspond to a generator with \(0 \leq\) PMIN \(<\) PMAX. A set of price limits can be specified via the lim struct, e.g. and offer price cap on real energy would be stored in lim.P.max_offer. Capacity offered above this price is considered to be withheld from the auction and is not included in the cost function produced. Bids must be in nonincreasing order of price and correspond to a generator with PMIN \(<\) PMAX \(\leq 0\) (see Section 6.4.2 on page 66). A lower limit can be set for bids in lim.P.min_bid. See help pricelimits for more information.

The data specified by a MATPOWER case file, with the gen and gencost matrices modified according to step 1 , are then used to run an OPF. A decommitment mechanism is used to shut down generators if doing so results in a smaller overall system cost (see Section 8).

\footnotetext{
\({ }^{50}\) See http://www.pserc.cornell.edu/powerweb/.
}

In step 3 the OPF solution is used to determine for each offer/bid block, how much was cleared and at what price. These values are returned in co and cb, which have the same structure as offers and bids. The mkt parameter is a struct used to specify a number of things about the market, including the type of auction to use, type of OPF (AC or DC) to use and the price limits.

There are two basic types of pricing options available through mkt. auction_type, discriminative pricing and uniform pricing. The various uniform pricing options are best explained in the context of an unconstrained lossless network. In this context, the allocation is identical to what one would get by creating bid and offer stacks and finding the intersection point. The nodal prices \(\left(\lambda_{P}\right)\) computed by the OPF and returned in bus (:, LAM_P) are all equal to the price of the marginal block. This is either the last accepted offer (LAO) or the last accepted bid (LAB), depending which is the marginal block (i.e. the one that is split by intersection of the offer and bid stacks). There is often a gap between the last accepted bid and the last accepted offer. Since any price within this range is acceptable to all buyers and sellers, we end up with a number of options for how to set the price, as listed in Table F-1.

Table F-1: Auction Types
\begin{tabular}{ccl}
\hline auction type & name & description \\
\hline 0 & discriminative & \begin{tabular}{l} 
price of each cleared offer (bid) is equal to the offered (bid) \\
price
\end{tabular} \\
1 & LAO & \begin{tabular}{l} 
uniform price equal to the last accepted offer \\
2
\end{tabular} \\
3 & FRO & uniform price equal to the first rejected offer \\
4 & LAB & uniform price equal to the last accepted bid \\
5 & FRB & uniform price equal to the first rejected bid \\
6 & second price & \begin{tabular}{l} 
uniform price equal to the offer/bid price of the marginal \\
unit \\
uniform price equal to min(FRO, LAB) if the marginal unit \\
is an offer, or max(FRB, LAO) if it is a bid
\end{tabular} \\
7 & split-the-difference & \begin{tabular}{l} 
uniform price equal to the average of the LAO and LAB \\
dual LAOB
\end{tabular} \\
\begin{tabular}{c} 
uniform price for sellers equal to LAO, for buyers equal to \\
LAB
\end{tabular} \\
\hline
\end{tabular}

Generalizing to a network with possible losses and congestion results in nodal prices \(\lambda_{P}\) which vary according to location. These \(\lambda_{P}\) values can be used to normalize all bids and offers to a reference location by multiplying by a locational scale factor. For bids and offers at bus \(i\), this scale factor is \(\lambda_{P}^{\mathrm{ref}} / \lambda_{P}^{i}\), where \(\lambda_{P}^{\mathrm{ref}}\) is the nodal price at the reference bus. The desired uniform pricing rule can then be applied to the adjusted offers and bids to get the appropriate uniform price at the reference
bus. This uniform price is then adjusted for location by dividing by the locational scale factor. The appropriate locationally adjusted uniform price is then used for all cleared bids and offers. \({ }^{51}\) The relationships between the OPF results and the pricing rules of the various uniform price auctions are described in detail in [46].

There are certain circumstances under which the price of a cleared offer determined by the above procedures can be less than the original offer price, such as when a generator is dispatched at its minimum generation limit, or greater than the price cap lim.P.max_cleared_offer. For this reason, all cleared offer prices are clipped to be greater than or equal to the offer price but less than or equal to lim.P.max_cleared_offer. Likewise, cleared bid prices are less than or equal to the bid price but greater than or equal to lim.P.min_cleared_bid.

\section*{F. 1 Handling Supply Shortfall}

In single sided markets, in order to handle situations where the offered capacity is insufficient to meet the demand under all of the other constraints, resulting in an infeasible OPF, we introduce the concept of emergency imports. We model an import as a fixed injection together with an equally sized dispatchable load which is bid in at a high price. Under normal circumstances, the two cancel each other and have no effect on the solution. Under supply shortage situations, the dispatchable load is not fully dispatched, resulting in a net injection at the bus, mimicking an import. When used in conjunction with the LAO pricing rule, the marginal load bid will not set the price if all offered capacity can be used.

\section*{F. 2 Example}

The case file lib/t/t_auction_case.m, used for this example, is a modified version of the 30 -bus system that has 9 generators, where the last three have negative PMIN to model the dispatchable loads.
- Six generators with three blocks of capacity each, offering as shown in Table F-2.
- Fixed load totaling 151.64 MW.
- Three dispatchable loads, bidding three blocks each as shown in Table F-3.

\footnotetext{
\({ }^{51}\) In versions of MATPOWER prior to 4.0 , the smart market code incorrectly shifted prices instead of scaling them, resulting in prices that, while falling within the offer/bid "gap" and therefore acceptable to all participants, did not necessarily correspond to the OPF solution.
}

Table F-2: Generator Offers
\begin{tabular}{cccc}
\hline Generator & \begin{tabular}{c} 
Block 1 \\
MW @ \$/MWh
\end{tabular} & \begin{tabular}{c} 
Block 2 \\
MW @ \$/MWh
\end{tabular} & \begin{tabular}{c} 
Block 3 \\
MW @ \(\$ /\) MWh
\end{tabular} \\
\hline 1 & \(12 @ \$ 20\) & \(24 @ \$ 50\) & \(24 @ \$ 60\) \\
2 & \(12 @ \$ 20\) & \(24 @ \$ 40\) & \(24 @ \$ 70\) \\
3 & \(12 @ \$ 20\) & \(24 @ \$ 42\) & \(24 @ \$ 80\) \\
4 & \(12 @ \$ 20\) & \(24 @ \$ 44\) & \(24 @ \$ 90\) \\
5 & \(12 @ \$ 20\) & \(24 @ \$ 46\) & \(24 @ \$ 75\) \\
6 & \(12 @ \$ 20\) & \(24 @ \$ 48\) & \(24 @ \$ 60\) \\
\hline
\end{tabular}

Table F-3: Load Bids
\begin{tabular}{cccc}
\hline Load & \begin{tabular}{c} 
Block 1 \\
MW @ \(\$ / \mathrm{MWh}\)
\end{tabular} & \begin{tabular}{c} 
Block 2 \\
MW @ \(\$ / \mathrm{MWh}\)
\end{tabular} & \begin{tabular}{c} 
Block 3 \\
MW @ \(\$ / \mathrm{MWh}\)
\end{tabular} \\
\hline 1 & \(10 @ \$ 100\) & \(10 @ \$ 70\) & \(10 @ \$ 60\) \\
2 & \(10 @ \$ 100\) & \(10 @ \$ 50\) & \(10 @ \$ 20\) \\
3 & \(10 @ \$ 100\) & \(10 @ \$ 60\) & \(10 @ \$ 50\) \\
\hline
\end{tabular}

To solve this case using an AC optimal power flow and a last accepted offer (LAO) pricing rule, we use:
```

mkt.OPF = 'AC';
mkt.auction_type = 1;

```
and set up the problem as follows:
```

mpc = loadcase('t_auction_case');
offers.P.qty = [ ...
12 24 24;
12 24 24;
12 24 24;
12 24 24;
12 24 24;
12 24 24 ];
offers.P.prc = [ ...
20 50 60;
20 40 70;
20 42 80;
20 44 90;
20 46 75;
20 48 60 ];
bids.P.qty = [ ...
10 10 10;
10 10 10;
10 10 10 ];
bids.P.prc = [ ...
100 70 60;
100 50 20;
100 60 50 ];
[mpc_out, co, cb, f, dispatch, success, et] = runmarket(mpc, offers, bids, mkt);

```

The resulting cleared offers and bids are:
```

>> co.P.qty
ans =
12.0000
12.0000
24.0000
0
12.0000
12.0000
12.0000
12.0000
>> co.P.prc
ans =
50.0000 50.0000 50.0000
50.2406 50.2406 50.2406
50.3368 50.3368 50.3368
51.0242 51.0242 51.0242
52.1697 52.1697 52.1697
52.9832 52.9832 52.9832
>> cb.P.qty
ans =
10.0000 10.0000 10.0000
10.0000
10.0000
10.0000
>> cb.P.prc
ans =
51.8207 51.8207 51.8207
54.0312 54.0312 54.0312
55.6208 55.6208 55.6208

```

In other words, the sales by generators and purchases by loads are as shown summarized in Tables F-4 and Tables F-5, respectively.

Table F-4: Generator Sales
\begin{tabular}{ccc}
\hline Generator & \begin{tabular}{c} 
Quantity Sold \\
\(M W\)
\end{tabular} & \begin{tabular}{c} 
Selling Price \\
\(\$ / M W h\)
\end{tabular} \\
\hline 1 & 35.3 & \(\$ 50.00\) \\
2 & 36.0 & \(\$ 50.24\) \\
3 & 36.0 & \(\$ 50.34\) \\
4 & 36.0 & \(\$ 51.02\) \\
5 & 36.0 & \(\$ 52.17\) \\
6 & 36.0 & \(\$ 52.98\) \\
\hline
\end{tabular}

Table F-5: Load Purchases
\begin{tabular}{ccc}
\hline Load & \begin{tabular}{c} 
Quantity Bought \\
\(M W\)
\end{tabular} & \begin{tabular}{c} 
Purchase Price \\
\(\$ / M W h\)
\end{tabular} \\
\hline 1 & 30.0 & \(\$ 51.82\) \\
2 & 10.0 & \(\$ 54.03\) \\
3 & 20.0 & \(\$ 55.62\) \\
\hline
\end{tabular}

\section*{F. 3 Smartmarket Files and Functions}

Table F-6: Smartmarket Files and Functions
\begin{tabular}{ll}
\hline name & description \\
\hline \begin{tabular}{ll} 
extras/smartmarket/ & \\
auction & clears set of bids and offers based on pricing rules and OPF results \\
case2off & generates quantity/price offers and bids from gen and gencost \\
idx_disp & named column index definitions for dispatch matrix \\
off2case & updates gen and gencost based on quantity/price offers and bids \\
pricelimits & fills in a struct with default values for offer and bid limits \\
printmkt & prints the market output \\
runmarket & top-level simulation function, runs the OPF-based smart market \\
runmkt & top-level simulation function, runs the OPF-based smart market \\
smartmkt & implements the smart market solver \\
SM_CHANGES & change history for the smart market software \\
\hline
\end{tabular} \\
\hline
\end{tabular}

\footnotetext{
* Deprecated. Will be removed in a subsequent version.
}

\section*{Appendix G Optional Packages}

There are a number of optional packages, not included in the Matpower distribution, that Matpower can utilize if they are installed in your Matlab path. Each of them is based on one or more MEX files pre-compiled for various platforms, some distributed by PSERC, others available from third parties, and each with their own terms of use.

\section*{G. 1 BPMPD_MEX - MEX interface for BPMPD}

BPMPD_MEX \([28,29]\) is a Matlab MEX interface to BPMPD, an interior point solver for quadratic programming developed by Csaba Mészáros at the MTA SZTAKI, Computer and Automation Research Institute, Hungarian Academy of Sciences, Budapest, Hungary. It can be used by Matpower's DC and LP-based OPF solvers and it improves the robustness of MINOPF. It is also useful outside of MATPOWER as a general-purpose QP/LP solver.

This MEX interface for BPMPD was coded by Carlos E. Murillo-Sánchez, while he was at Cornell University. It does not provide all of the functionality of BPMPD, however. In particular, the stand-alone BPMPD program is designed to read and write results and data from MPS and QPS format files, but this MEX version does not implement reading data from these files into Matlab.

The current version of the MEX interface is based on version 2.21 of the BPMPD solver, implemented in Fortran.

Builds are available for Linux (32-bit), Mac OS X (PPC, Intel 32-bit) and Windows (32-bit) at http://www.pserc. cornell.edu/bpmpd/.

When installed BPMPD_MEX can be selected as the solver for DC OPFs by setting the opf.dc.solver option to 'BPMPD'. It can also be used to solve general LP and QP problems via MATPOWER's common QP solver interface qps matpower with the algorithm option set to 'BPMPD' (or 100 for backward compatibility), or by calling qps_bpmpd directly.

\section*{G. 2 CLP - COIN-OR Linear Programming}

The CLP [35] (COIN-OR Linear Programming) solver is an open-source linear programming solver written in C++ by John Forrest. It can solve both linear programming (LP) and quadratic programming (QP) problems. It is primarily meant to be used as a callable library, but a basic, stand-alone executable version exists as well. It is available from the COIN-OR initiative at http://www.coin-or.org/projects/ Clp.xml.

To use CLP with Matpower, a MEX interface is required \({ }^{52}\). For Microsoft Windows users, a pre-compiled MEX version of CLP (and numerous other solvers, such as GLPK and Ipopt) are easily installable as part of the OPTI Toolbox \({ }^{53}\) [47]

With the Matlab interface to CLP installed, it can be used to solve DC OPF problems by setting the opf.dc.solver option equal to 'CLP'. The solution algorithms and other CLP parameters can be set directly via Matpower's clp.opts option. A "CLP user options" function can also be specified via clp.opt_fname to override the defaults for any of the many CLP parameters. See help clp for details. See Table C-8 for a summary of the CLP-related Matpower options.

CLP can also be used to solve general LP and QP problems via Matpower's common QP solver interface qps matpower with the algorithm option set to 'CLP', or by calling qps_clp directly.

\section*{G. 3 CPLEX - High-performance LP and QP Solvers}

The IBM ILOG CPLEX Optimizer, or simply CPLEX, is a collection of optimization tools that includes high-performance solvers for large-scale linear programming (LP) and quadratic programming (QP) problems, among others. More information is available at http://www.ibm.com/software/integration/optimization/ cplex-optimizer/.

Although CPLEX is a commercial package, at the time of this writing the full version is available to academics at no charge through the IBM Academic Initiative program for teaching and non-commercial research. See http://www.ibm.com/ support/docview.wss?uid=swg21419058 for more details.

When the Matlab interface to CPLEX is installed, the CPLEX LP and QP solvers, cplexlp and cplexqp, can be used to solve DC OPF problems by setting the opf.dc.solver option equal to 'CPLEX'. The solution algorithms can be controlled by Matpower's cplex.lpmethod and cplex.qpmethod options. See Table C-9 for a summary of the CPLEX-related MATPOWER options. A "CPLEX user options" function can also be specified via cplex.opt_fname to override the defaults for any of the many CPLEX parameters. See help cplex_options and the "Parameters of

\footnotetext{
\({ }^{52}\) According to David Gleich at http://web.stanford.edu/~dgleich/notebook/2009/03/ coinor_clop_for_matlab.html, there was a MATLAB MEX interface to CLP written by Johan Lofberg and available (at some point in the past) at http://control.ee.ethz.ch/~joloef/ mexclp.zip. Unfortunately, at the time of this writing, it seems it is no longer available there, but Davide Barcelli makes some precompiled MEX files for some platforms available here http://www.dii.unisi.it/~barcelli/software.php, and the ZIP file linked as Clp 1.14 .3 contains the MEX source as well as a clp.m wrapper function with some rudimentary documentation.
\({ }^{53}\) The OPTI Toolbox is available from http://www.i2c2.aut.ac.nz/Wiki/OPTI/.
}

CPLEX" section of the CPLEX documentation at http://www.ibm.com/support/ knowledgecenter/SSSA5P for details.

It can also be used to solve general LP and QP problems via Matpower's common QP solver interface qps_matpower, or MILP and MIQP problems via miqps matpower, with the algorithm option set to 'CPLEX' (or 500 for backward compatibility), or by calling qps_cplex or miqps_cplex directly.

\section*{G. 4 GLPK - GNU Linear Programming Kit}

The GLPK [34] (GNU Linear Programming Kit) package is intended for solving large-scale linear programming (LP), mixed integer programming (MIP), and other related problems. It is a set of routines written in ANSI C and organized in the form of a callable library.

To use GLPK with Matpower, a MEX interface is required \({ }^{54}\). For Microsoft Windows users, a pre-compiled MEX version of GLPK (and numerous other solvers, such as CLP and Ipopt) are easily installable as part of the OPTI Toolbox \({ }^{55}\) [47].

When GLPK is installed, either as part of Octave or with a MEX interface for Matlab, it can be used to solve DC OPF problems that do not include any quadratic costs by setting the opf.dc.solver option equal to 'GLPK'. The solution algorithms and other GLPK parameters can be set directly via MATPOWER's glpk. opts option. A "GLPK user options" function can also be specified via glpk.opt_fname to override the defaults for any of the many GLPK parameters. See help glpk_options and the parameters section the GLPK documentation at http://www.gnu.org/software/ octave/doc/interpreter/Linear-Programming.html for details. See Table C-11 for a summary of the GLPK-related Matpower options.

GLPK can also be used to solve general LP problems via MatPower's common QP solver interface qps matpower, or MILP problems via miqps matpower, with the algorithm option set to 'GLPK', or by calling qps_glpk or miqps_glpk directly.

\section*{G. 5 Gurobi - High-performance LP and QP Solvers}

Gurobi [33] is a collection of optimization tools that includes high-performance solvers for large-scale linear programming (LP) and quadratic programming (QP) problems,

\footnotetext{
\({ }^{54}\) The http://glpkmex.sourceforge.net site and Davide Barcelli's page http://www.dii. unisi.it/~barcelli/software.php may be useful in obtaining the MEX source or pre-compiled binaries for Mac or Linux platforms.
\({ }^{55}\) The OPTI Toolbox is available from http://www.i2c2.aut.ac.nz/Wiki/OPTI/.
}
among others. The project was started by some former CPLEX developers. More information is available at http://www.gurobi.com/.

Although Gurobi is a commercial package, at the time of this writing their is a free academic license available. See http://www.gurobi.com/html/academic.html for more details.

Beginning with version 5.0.0, Gurobi includes a native Matlab interface, which is supported in MATPOWER version 4.2 and above. \({ }^{56}\)

When Gurobi is installed, it can be used to solve DC OPF problems by setting the opf.dc.solver option equal to 'GUROBI'. The solution algorithms can be controlled by Matpower's gurobi.method option. See Table C-12 for a summary of the Gurobi-related Matpower options. A "Gurobi user options" function can also be specified via gurobi.opt_fname to override the defaults for any of the many Gurobi parameters. See help gurobi_options and the "Parameters" section of the "Gurobi Optimizer Reference Manual" at http://www.gurobi.com/documentation/6.0/refman/ parameters.html for details.

It can also be used to solve general LP and QP problems via MATPOWER's common QP solver interface qps_matpower, or MILP and MIQP problems via miqps matpower, with the algorithm option set to 'GUROBI' (or 700 for backward compatibility), or by calling qps_gurobi or miqps_gurobi directly.

\section*{G. 6 IPOPT - Interior Point Optimizer}

Ipopt [49] (Interior Point OPTimizer, pronounced I-P-Opt) is a software package for large-scale nonlinear optimization. It is is written in \(\mathrm{C}++\) and is released as open source code under the Common Public License (CPL). It is available from the COIN-OR initiative at http://www.coin-or.org/projects/Ipopt.xml. The code has been written by Carl Laird and Andreas Wächter, who is the COIN project leader for Ipopt.

Matpower requires the Matlab MEX interface to Ipopt, which is included in the Ipopt source distribution, but must be built separately. Additional information on the MEX interface is available at https://projects.coin-or.org/Ipopt/wiki/ MatlabInterface. Please consult the Ipopt documentation, web-site and mailing lists for help in building and installing the Ipopt Matlab interface. This interface uses callbacks to Matlab functions to evaluate the objective function and its gradient, the constraint values and Jacobian, and the Hessian of the Lagrangian.

\footnotetext{
\({ }^{56}\) MATPOWER version 4.1 supported Gurobi version 4.x, which required a free third-party MatLab MEX interface [48], available at http://www. convexoptimization.com/wikimization/ index.php/Gurobi_mex.
}

Precompiled MEX binaries for a high-performance version of Ipopt, using the PARDISO linear solver \([50,51]\), are available from the PARDISO project \({ }^{57}\). At the time of this writing, these are the highest performing solvers available to Matpower for very large scale AC OPF problems. For Microsoft Windows users, a pre-compiled MEX version of IPOPt (and numerous other solvers, such as CLP and GLPK) are easily installable as part of the OPTI Toolbox \({ }^{58}\) [47].

When installed, Ipopt can be used by Matpower to solve both AC and DC OPF problems by setting the opf.ac.solver or opf.dc.solver options, respectively, equal to 'IPOPT'. See Table C-13 for a summary of the IPOPT-related Matpower options. The many algorithm options can be set by creating an "Ipopt user options" function and specifying it via ipopt.opt_fname to override the defaults set by ipopt_options. See help ipopt_options and the options reference section of the IPOPT documentation at http://www. coin-or.org/Ipopt/documentation/for details.

It can also be used to solve general LP and QP problems via Matpower's common QP solver interface qps_matpower with the algorithm option set to 'IPOPT' (or 400 for backward compatibility), or by calling qps_ipopt directly.

\section*{G. 7 KNITRO - Non-Linear Programming Solver}

KNITRO [32] is a general purpose optimization solver specializing in nonlinear problems, available from Ziena Optimization, LLC. As of version 9, KNITRO includes a native Matlab interface, knitromatlab \({ }^{59}\). More information is available at http://www.ziena.com/ and http://www.ziena.com/knitromatlab.htm.

Although KNITRO is a commercial package, at the time of this writing there is a free academic license available, with details on their download page.

When installed, KNITRO's MATLAB interface function, knitromatlab or ktrlink, can be used by Matpower to solve AC OPF problems by simply setting the opf.ac.solver option to 'KNITRO'. See Table C-14 for a summary of KNITRO-related Matpower options. The knitromatlab function uses callbacks to Matlab functions to evaluate the objective function and its gradient, the constraint values and Jacobian, and the Hessian of the Lagrangian.

KNITRO options can be controlled directly by creating a standard KNITRO op-

\footnotetext{
\({ }^{57}\) See http://www.pardiso-project.org/ for the download links.
\({ }^{58}\) The OPTI Toolbox is available from http://www.i2c2.aut.ac.nz/Wiki/OPTI/.
\({ }^{59}\) Earlier versions required the Matlab Optimization Toolbox from The MathWorks, which includes an interface to the KNITRO libraries called ktrlink, but the libraries themselves still had to be acquired directly from Ziena Optimization, LLC.
}
tions file in your working directory and specifying it via the knitro.opt_fname (or, for backward compatibility, naming it knitro_user_options_n.txt and setting MATPOWER's knitro.opt option to \(n\), where \(n\) is some positive integer value). See the KNITRO user manuals at http://www.ziena.com/documentation.htm for details on the available options.

\section*{G. 8 MINOPF - AC OPF Solver Based on MINOS}

MINOPF [30] is a MINOS-based optimal power flow solver for use with Matpower. It is for educational and research use only. MINOS [31] is a legacy Fortran-based software package, developed at the Systems Optimization Laboratory at Stanford University, for solving large-scale optimization problems.

While MINOPF is often MATPOWER's fastest AC OPF solver on small problems, as of Matpower 4, it no longer becomes the default AC OPF solver when it is installed. It can be selected manually by setting the opf.ac.solver option to 'MINOPF' (see help mpoption for details).

Builds are available for Linux (32-bit), Mac OS X (PPC, Intel 32-bit) and Windows (32-bit) at http://www.pserc.cornell.edu/minopf/.

\section*{G. 9 MOSEK - High-performance LP and QP Solvers}

MOSEK is a collection of optimization tools that includes high-performance solvers for large-scale linear programming (LP) and quadratic programming (QP) problems, among others. More information is available at http://www.mosek.com/.

Although MOSEK is a commercial package, at the time of this writing there is a free academic license available. See http://mosek.com/resources/academic-license/ for more details.

When the Matlab interface to MOSEK is installed, the MOSEK LP and QP solvers can be used to solve DC OPF problems by setting the opf.dc.solver option equal to 'MOSEK'. The solution algorithm for LP problems can be controlled by Matpower's mosek.lp_alg option. See Table C-16 for other MOSEK-related MATPOWER options. A "MOSEK user options" function can also be specified via mosek.opt_fname to override the defaults for any of the many MOSEK parameters. For details see help mosek_options and the "Parameters" reference in "The MOSEK optimization toolbox for MATLAB manual" at http://docs.mosek.com/ \(7.1 /\) toolbox/Parameters.html. You may also find it helpful to use the symbolic constants defined by mosek_symbcon.

It can also be used to solve general LP and QP problems via MATPOWER's common QP solver interface qps_matpower, or MILP and MIQP problems via miqps_matpower, with the algorithm option set to 'MOSEK' (or 600 for backward compatibility), or by calling qps_mosek or miqps_mosek directly.

\section*{G. 10 Optimization Toolbox - LP, QP, NLP and MILP Solvers}

Matlab's Optimization Toolbox \([27,52]\), available from The MathWorks, provides a number of high-performance solvers that MATPOWER can take advantage of.

It includes fmincon for nonlinear programming problems (NLP), and linprog and quadprog for linear programming (LP) and quadratic programming (QP) problems, respectively. For mixed-integer linear programs (MILP), it provides intlingprog. Each solver implements a number of different solution algorithms. More information is available from The MathWorks, Inc. at http://www.mathworks.com/.

When available, the Optimization Toolbox solvers can be used to solve AC or DC OPF problems by setting the opf.ac.solver or opf.dc.solver options, respectively, equal to 'OT'.

The solution algorithm used by fmincon for NLP problems can be controlled by Matpower's fmincon.alg option. See Table C-10 for other Matpower options related to fmincon. For linprog, quadprog and intlingprog, the corresponding MATPOWER option can be used to pass in native Optimization Toolbox options directly to the solver. For example, to set the LP solver to use a dual simplex method, simply set Matpower's 'linprog.Algorithm' option to 'dual-simplex'. For details on the full set of Optimization Toolbox options, please refer to their documentation. \({ }^{60}\)

The Optimization Toolbox can also be used to solve general LP and QP problems via MATPOWER's common QP solver interface qps_matpower, or MILP problems via miqps matpower, with the algorithm option set to 'OT', or by calling qps_ot or miqps_ot directly.

\section*{G. 11 PARDISO - Parallel Sparse Direct and Multi-Recursive Iterative Linear Solvers}

The PARDISO package is a thread-safe, high-performance, robust, memory efficient and easy to use software for solving large sparse symmetric and non-symmetric linear systems of equations on shared-memory and distributed-memory multiprocessor systems [50,51]. More information is available at http://www.pardiso-project.org.

\footnotetext{
\({ }^{60}\) See http://www.mathworks.com/help/optim/ and [52].
}

When the Matlab interface to PARDISO is installed, PARDISO's solvers can be used to replace the built-in \(\backslash\) operator for solving for the Newton update step in Matpower's default primal-dual interior point solver (MIPS) by setting the mips.linsolver option equal to 'PARDISO'. The mplinsolve function can also be called directly to solve \(A x=b\) problems via PARDISO or the built-in solver, depending on the arguments supplied. This interface also gives access to the full range of PARDISO's options. For details, see help mplinsolve and the PARDISO User's Manual at http://www.pardiso-project.org/manual/manual.pdf.

When solving very large AC optimal power flow problems with MIPS, selecting PARDISO as the linear solver can often dramtically improve both computation time and memory use.

Also note that precompiled MEX binaries for a high-performance version of Ipopt, using the PARDISO linear solver, are available. Refer to Section G. 6 for more details.

\section*{G. 12 SDP_PF - Applications of a Semidefinite Programming Relaxation of the Power Flow Equations}

This package is contributed by Dan Molzahn and is currently included with MatPOWER in the <MATPOWER>/extras/sdp_pf directory. Complete documentation is available in \(<\) MATPOWER \(>/\) extras/sdp_pf/documentation directory, and especially in the file sdp_pf_documentation.pdf.

\section*{G. 13 TSPOPF - Three AC OPF Solvers by H. Wang}

TSPOPF [25] is a collection of three high performance AC optimal power flow solvers for use with Matpower. The three solvers are:
- PDIPM - primal/dual interior point method
- SCPDIPM - step-controlled primal/dual interior point method
- TRALM - trust region based augmented Lagrangian method

The algorithms are described in \([26,41]\). The first two are essentially C-language implementations of the algorithms used by MIPS (see Appendix A), with the exception that the step-controlled version in TSPOPF also includes a cost smoothing technique in place of the constrained-cost variable (CCV) approach for handling piece-wise linear costs.

The PDIPM in particular is significantly faster for large systems than any previous Matpower AC OPF solver, including MINOPF. When TSPOPF is installed, the PDIPM solver becomes the default optimal power flow solver for Matpower. Additional options for TSPOPF can be set using mpoption (see help mpoption for details).

Builds are available for Linux (32-bit, 64-bit), Mac OS X (PPC, Intel 32-bit, Intel 64-bit) and Windows (32-bit, 64-bit) at http://www.pserc.cornell.edu/tspopf/.

\section*{Appendix H Release History}

The full release history can be found in docs/CHANGES or online at http://www. pserc.cornell.edu/matpower/CHANGES.txt.

\section*{H. 1 Pre 1.0 - released Jun 25, 1997}

Work on Matpower began in 1996, with an optimal power flow based on Matlab's constr function.

\section*{H. 2 Version 1.0 - released Sep 17, 1997}

The Matpower 1.0 User's Manual is available online. \({ }^{61}\)

\section*{New Features}
- Successive LP-based optimal power flow solver, by Deqiang Gan.
- Automatic conversion between possibly non-consecutive external bus numbers and consecutive internal bus numbers.
- IEEE CDF to MATPOWER case conversion script, by Deqiang Gan.

\section*{Other Changes}
- Top-level scripts converted to functions that take case file as input.
- Updated case file format. Generator costs moved to separate gencost table.

\section*{H. 3 Version 1.0.1 - released Sep 19, 1997}

Changes
- Bug fixes.

\footnotetext{
\({ }^{61}\) http://www.pserc.cornell.edu/matpower/docs/MATPOWER-manual-1.pdf
}

\section*{H. 4 Version 2.0 - released Dec 24, 1997}

The Matpower 2.0 User's Manual is available online. \({ }^{62}\)

\section*{New Features}
- greatly enhanced output options
- fast-decoupled power flow
- optional costs for reactive power generation in OPF
- consolidated parameters in Matpower options vector

\section*{Other Changes}
- optimized building of \(Y_{\text {bus }}\) and Jacobian matrices (much faster on large systems, especially in Matlab 5)
- highly improved unit decommitment algorithm
- completely rewritten LP-based OPF solver
- various bug fixes

\footnotetext{
\({ }^{62}\) http://www.pserc.cornell.edu/matpower/docs/MATPOWER-manual-2.0.pdf
}

\section*{H. 5 Version 3.0 - released Feb 14, 2005}

The Matpower 3.0 User's Manual is available online. \({ }^{63}\)

\section*{New Features}
- Compatibility with Matlab 7 and Optimization Toolbox 3.
- DC power flow and DC OPF solvers added.
- Option to enforce generator reactive power limits in AC power flow solution.
- Gauss-Seidel power flow solver added.
- Support for MINOS-based OPF solver added (separate package, see http://www.pserc.cornell.edu/minopf/ for more details)
- Multiple generators at a single bus.
- Saving of solved cases as M-files or MAT-files.
- Loading of input data from M-files, MAT-files, or structs.
- Improved decommitment algorithm.
- Added a very incomplete test suite.
- Handling of dispatchable loads in OPF, modeled as negative generators with constant power factor constraint.

\section*{Bugs Fixed}
- Phase shifters shifted the wrong direction.
- Minor fixes to IEEE CDF to Matpower format conversion (reported by D. Devaraj and Venkat)
- Flows on out-of-service lines were not being zeroed out. (reported by Ramazan Caglar)
- Reported total inter-tie flow values and area export values were incorrect.
- Several other bugs in solution printouts.

\footnotetext{
\({ }^{63}\) http://www.pserc.cornell.edu/matpower/docs/MATPOWER-manual-3.0.pdf
}

\section*{H. 6 Version 3.2 - released Sep 21, 2007}

The Matpower 3.2 User's Manual is available online. \({ }^{64}\)

\section*{New Features}
- AC OPF formulation enhancements
- new generalized cost model
- piece-wise linear generator PQ capability curves
- branch angle difference constraints
- simplified interface for specifying additional linear constraints
- option to use current magnitude for line flow limits (set OPF_FLOW_LIM to 2 , fmincopf solver only)
- AC OPF solvers
- support for TSPOPF, a new optional package of three OPF solvers, implemented as C MEX files, suitable for large scale systems
- ability to specify initial value and bounds on user variables \(z\)
- New (v. 2) case file format
- all data in a single struct
- generator PQ capability curves
- generator ramp rates
- branch angle difference limits
- New function makePTDF to build DC PDTF matrix
- Added 5 larger scale ( \(>2000\) bus) cases for Polish system (thanks to Roman Korab).
- Improved identification of binding constraints in printout.
- Many new tests in test suite.

\footnotetext{
\({ }^{64}\) http://www.pserc.cornell.edu/matpower/docs/MATPOWER-manual-3.2.pdf
}

\section*{Bugs Fixed}
- Phase shifters shifted the wrong direction, again (v. 2 had it right).
- Fixed bug in pfsoln which caused incorrect value for reactive generation when \(Q_{\text {min }}=Q_{\max }\) for all generators at a bus in power flow solution.

\section*{Incompatible Changes}
- User supplied \(A\) matrix for general linear constraints in OPF no longer includes columns for helper variables for piecewise linear gen costs, and now requires columns for all \(x\) (OPF) variables.
- Changed the sign convention used for phase shifters to be consistent with PTI, PowerWorld, PSAT, etc. E.g. A phase shift of 10 deg now means the voltage at the "to" end is delayed by 10 degrees.
- Name of option 24 in mpoption changed from OPF_P_LINE_LIM to OPF_FLOW_LIM.

\section*{H. 7 Version 4.0 - released Feb 7, 2011}

The Matpower 4.0 User's Manual is available online. \({ }^{65}\)

\section*{New Features}
- Licensed under the GNU General Public License (GPL).
- Added compatibility with GNU Octave, a free, open-source Matlab clone.
- Extensive OPF enhancements:
- Generalized, extensible OPF formulation applies to all solvers (AC and DC).
- Improved method for modifying OPF formulation and output via a new user-defined callback function mechanism.
- Option to co-optimize reserves based on fixed zonal reserve requirements, implemented using new callback function mechanism.
- Option to include interface flow limits (based on DC model flows), implemented using new callback function mechanism.
- New high performance OPF solvers:
- MIPS (Matpower Interior Point Solver), a new a pure-Matlab implementation of the primal-dual interior point methods from the optional package TSPOPF. MIPS is suitable for large systems and is used as MATPOWER's default solver for AC and DC OPF problems if no other optional solvers are installed. To select MIPS explicitly, use OPF_ALG \(=560 / 565\) and OPF_ALG_DC \(=200 / 250\) for AC and DC OPF, respectively. MIPS can also be used independently of MATPOWER as a solver for general nonlinear constrained optimization problems.
- Support for the Ipopt interior point optimizer for large scale nonlinear optimization. Use OPF_ALG \(=580\) and OPF_ALG_DC \(=400\) for AC and DC OPF, respectively. Requires the Matlab MEX interface for Ipopt, available from https://projects.coin-or.org/Ipopt/.
- Support for CPLEX to solve LP and QP problems. Set option OPF_ALG_DC \(=500\) to use CPLEX to solve the DC OPF. Requires the Matlab interface to

\footnotetext{
\({ }^{65}\) http://www.pserc.cornell.edu/matpower/docs/MATPOWER-manual-4.0.pdf
}

CPLEX, available from http://www.ibm.com/software/integration/ optimization/cplex-optimizer/. See help mpoption for more CPLEX options.
- Support for MOSEK to solve LP and QP problems. Set option OPF_ALG_DC = 600 to use MOSEK to solve the DC OPF. Requires the Matlab interface to MOSEK, available from http://www.mosek.com/. See help mpoption for more MOSEK options.
- Updated support for MathWorks' Optimization Toolbox solvers, fmincon(), linprog() and quadprog().
- Improved documentation:
- New, rewritten User's Manual (docs/manual.pdf).
- Two new Tech Notes, available from Matpower home page.
* Uniform Price Auctions and Optimal Power Flow
* AC Power Flows, Generalized OPF Costs and their Derivatives using Complex Matrix Notation
- Help text updates to more closely match MathWorks conventions.
- New functions:
- load2disp() converts from fixed to dispatchable loads.
- makeJac() forms the power flow Jacobian. Optionally returns the system admittance matrices too.
- makeLODF () computes line outage distribution factors.
- modcost() shifts/scales generator cost functions.
- qps_matpower() provides a consistent, unified interface to all of MatPOWER's available QP/LP solvers, serving as a single wrapper around qps_bpmpd(), qps_cplex(), qps_ipopt(), qps_mips(), and qps_ot() (Optimization Toolbox, i.e. quadprog(), linprog()).
- scale_load() conveniently modifies multiple loads.
- total_load() retreives total load for the entire system, a specific zone or bus, with options to include fixed loads, dispatchable loads or both.
- Option to return full power flow or OPF solution in a single results struct, which is a superset of the input case struct.
- Ability to read and save generalized OPF user constraints, costs and variable limits as well as other user data in case struct.
- Numerous performance optimizations for large scale systems.
- Perl script psse2matpower for converting PSS/E data files to Matpower case format.
- Deprecated areas data matrix (was not being used).
- Many new tests in test suite.

\section*{Bugs Fixed}
- Auction code in extras/smartmarket in all previous versions contained a design error which has been fixed. Prices are now scaled instead of shifted when modified according to specified pricing rule (e.g. LAO, FRO, LAB, FRB, split-the-difference, etc.). Auctions with both real and reactive offers/bids must be type 0 or 5 , type \(1=\) LAO is no longer allowed.
- Branch power flow limits could be violated when using the option OPF_FLOW_LIM \(=1\).

\section*{Incompatible Changes}
- Renamed functions used to compute AC OPF cost, constraints and hessian, since they are used by more than fmincon:
```

costfmin }->\mathrm{ opf_costfcn
consfmin }->\mathrm{ opf_consfcn
hessfmin }->\mathrm{ opf_hessfcn

```
- Input/output arguments to uopf() are now consistent with opf().
- dAbr_dV() now gives partial derivatives of the squared magnitudes of flows w.r.t. \(V\), as opposed to the magnitudes.

\section*{H. 8 Version 4.1 - released Dec 14, 2011}

The Matpower 4.1 User's Manual is available online. \({ }^{66}\)

\section*{New Features}
- More new high performance OPF solvers:
- Support for the KNITRO interior point optimizer for large scale nonlinear optimization. Use OPF_ALG \(=600\) for to select KNITRO to solve the AC OPF. Requires the Matlab Optimization Toolbox and a license for KNITRO, available from http://www.ziena.com/. See help mpoption for more KNITRO options.
- Support for Gurobi to solve LP and QP problems. Set option OPF_ALG_DC = 700 to use Gurobi to solve the DC OPF. Requires Gurobi (http://www. gurobi.com/) and the Gurobi MEX interface (http://www. convexoptimization. com/wikimization/index.php/Gurobi_mex). See help mpoption for more Gurobi options.
- Updated for compatibility with CPLEX 12.3.
- Changed options so that fmincon uses its interior-point solver by default. Much faster on larger systems.
- Support for basic modeling of DC transmission lines.
- New case files with more recent versions of Polish system.
- Power flow can handle networks with islands.

\section*{Bugs Fixed}
- Computation of quadratic user-defined costs had a potentially fatal error. Thanks to Stefanos Delikaraoglou for finding this.
- Calculation of reserve prices in toggle_reserves() had an error.

\section*{Incompatible Changes}
- Optional packages TSPOPF and MINOPF must be updated to latest versions.

\footnotetext{
\({ }^{66}\) http://www.pserc.cornell.edu/matpower/docs/MATPOWER-manual-4.1.pdf
}

\section*{H. 9 Version 5.0 - released Dec 17, 2014}

The Matpower 5.0 User's Manual is available online. \({ }^{67}\)

\section*{New Features}
- Continuation power flow with tangent predictor and Newton method corrector, based on code contributed by Shrirang Abhyankar and Alex Flueck.
- SDP_PF, a set of applications of a semidefinite programming relaxation of the power flow equations, contributed by Dan Molzahn (see extras/sdp_pf):
- Globally optimal AC OPF solver (under certain conditions).
- Functions to check sufficient conditions for:
* global optimality of OPF solution
* insolvability of power flow equations
- PSS/E RAW data conversion to MATPOWER case format (experimental) based on code contributed by Yujia Zhu.
- Brand new extensible Matpower options architecture based on options struct instead of options vector.
- Utility routines to check network connectivity and handle islands and isolated buses.
- New extension implementing DC OPF branch flow soft limits. See help toggle_softlims for details.
- New and updated support for 3rd party solvers:
- CPLEX 12.6
- GLPK
- Gurobi 5.x
- Ipopt 3.11.x
- KNITRO 9.x.x
- Optimization Toolbox 7.1
\({ }^{67}\) http://www.pserc. cornell.edu/matpower/docs/MATPOWER-manual-5.0.pdf
- Numerous performance enhancements.
- New functions:
- runcpf() for continuation power flow.
- case_info() for summarizing system information, including network connectivity.
- extract_islands() to extract a network island into a separate MatPOWER case.
- find_islands() to detect network islands.
- @opt_model/describe_idx() to identify variable, constraint or cost row indices to aid in debugging.
- margcost() for computing the marginal cost of generation.
- psse2mpc() to convert PSS/E RAW data into MATPOWER case format.
- get_losses() to compute branch series losses and reactive charging injections and derivatives as functions of bus voltages.
- New experimental functions in extras/misc for computing loss factors, checking feasibility of solutions, converting losses to negative bus injections and modifying an OPF problem to make it feasible.
- Added case5.m, a 5-bus, 5-generator example case from Rui Bo.
- New options:
- scale_load() can scale corresponding gencost for dispatchable loads.
- makeJac() can return full Jacobian instead of reduced version used in Newton power flow updates.
- modcost() can accept a vector of shift/scale factors.
- total_load() can return actual or nominal values for dispatchable loads.
- runpf(), runopf(), etc. can send pretty-printed output to file without also sending it to the screen.
- out.suppress_detail option suppresses all output except system summary (on by default for large cases).
- opf.init_from_mpc option forces some solvers to use user-supplied starting point.
- MIPS 1.1 includes many new user-settable options.
- Reimplementated @opf model class as sub-class of the new @opt_model class, which supports indexed named sets of variables, constraints and costs.
- Many new tests in test suite.

\section*{Bugs Fixed}
- Running a power flow for a case with DC lines but no gencost no longer causes an error.
- Fixed a bug in runpf() where it was using the wrong initial voltage magnitude for generator buses marked as PQ buses. Power flow of solved case was not converging in zero iterations as expected.
- Fixed fatal bug in MIPS for unconstrained, scalar problems. Thanks to Han Na Gwon.
- Fixed a bug in int2ext() where converting a case to internal ordering before calling runpf() or runopf() could result in a fatal error due to mismatched number of columns in internal and external versions of data matrices. Thanks to Nasiruzzaman and Shiyang Li for reporting and detailing the issue.
- DC OPF now correctly sets voltage magnitudes to 1 p.u. in results.
- Fixed a bug in MIPS where a near-singular matrix could produce an extremely large Newton step, resulting in incorrectly satisfying the relative feasibility criterion for successful termination.
- Improved the starting point created for Ipopt, KNITRO and MIPS for variables that are only bounded on one side.
- Fixed bug in savecase() where the function name mistakenly included the path when the fname input included a path.
- Fixed bugs in runpf () related to enforcing generator reactive power limits when all generators violate limits or when the slack bus is converted to PQ.
- Fixed crash when using KNITRO to solve cases with all lines unconstrained.
- Fixed memory issue resulting from nested om fields when repeatedly running an OPF using the results of a previous OPF as input. Thanks to Carlos MurilloSánchez.
- Fixed fatal error when uopf() shuts down all gens attempting to satisfy \(P_{\min }\) limits.
- Reactive power output of multiple generators at a PQ bus no longer get reallocated when running a power flow.
- Fixed a bug in savecase() where a gencost matrix with extra columns of zeros resulted in a corrupted Matpower case file.
- Fixed bug in runpf() that caused a crash for cases with pf.enforce_q_lims turned on and exactly two Q limit violations, one Qmax and one Qmin. Thanks to Jose Luis Marín.

\section*{Incompatible Changes}
- Optional packages TSPOPF and MINOPF must be updated to latest versions.
- Renamed \(\operatorname{cdf} 2\) matp() to \(\operatorname{cdf} 2 \mathrm{mpc}()\) and updated the interface to be consistent with psse2mpc().
- Removed ot_opts field, replaced with linprog_opts and quadprog_opts fields in the opt argument to qps_matpower() and qps_ot().
- The name of the mips() option used to specify the maximum number of stepsize reductions with step_control on was changed from max_red to sc.red_it for consistency with other Matpower options.
- Removed max_it option from qps_matpower() (and friends) args. Use algorithm specific options to control iteration limits.
- Changed behavior of branch angle difference limits so that 0 is interpreted as unbounded only if both ANGMIN and ANGMAX are zero.
- In results struct returned by an OPF, the value of results.raw.output.alg is now a string, not an old-style numeric alg code.
- Removed:
- Support for Matlab 6.x.
- Support for constr () and successive LP-based OPF solvers.
- Support for Gurobi 4.x/gurobi mex() interface.
- extras/cpf, replaced by runcpf().
- extras/psse2matpower, replaced by psse2mpc().

\section*{H. 10 Version 5.1 - released Mar 20, 2015}

The Matpower 5.1 User's Manual is available online. \({ }^{68}\)

\section*{New License}
- Switched to the more permissive 3-clause BSD license from the previously used GNU General Public License (GPL) v3.0.

\section*{New Case Files}
- Added four new case files, ranging from 89 up to 9421 buses, representing parts of the European high voltage transmission network, stemming from the Pan European Grid Advanced Simulation and State Estimation (PEGASE) project. Thanks to Cédric Josz and colleagues from the French Transmission System Operator.

\section*{New Documentation}
- Added an online function reference to the website at http://www.pserc. cornell.edu/matpower/docs/ref/.

\section*{New Features}
- Added support for using PARDISO (http://www.pardiso-project.org/) as linear solver for computing interior-point update steps in MIPS, resulting in dramatic improvements in computation time and memory use for very largescale problems.
- Added support for LP/QP solver CLP (COIN_OR Linear Programming, http: //www.coin-or.org/projects/Clp.xml). Use opf.dc.solver option 'CLP' or qps_clp().
- Added support for OPTI Toolbox (http://www.i2c2.aut.ac.nz/Wiki/OPTI/) versions of CLP, GLPK and IPOPT solvers, providing a very simple installation path for some free high-performance solvers on Windows platforms.
- Network reduction toolbox for creating smaller approximate network equivalents from a larger original case file. Contributed by Yujia Zhu and Daniel Tylavsky.

\footnotetext{
\({ }^{68}\) http://www.pserc.cornell.edu/matpower/docs/MATPOWER-manual-5.1.pdf
}
- Added unified interface to various solvers for mixed-integer linear and quadratic programming (MILP/MIQP) problems.
- Major update to have_fen(), which now determines and caches version numbers and release dates for optional packages, and includes ability to toggle the availability of optional functionality.
- New and updated support for 3rd party solvers:
- High-performance Ipopt-PARDISO solver builds from the PARDISO Project http://www. pardiso-project.org (at time of release this is MATPOWER's highest performing solver for very large scale AC OPF problems)
- OPTI Toolbox versions of CLP, GLPK, Ipopt
- CLP
- Gurobi 6.x
- KNITRO 9.1
- MOSEK 7.1
- Optimization Toolbox 7.2
* dual-simplex algorithm for linprog()
* intlinprog() for MILP
- New functions:
- mplinsolve() provides unified interface for linear system solvers, including PARDISO and built-in backslash operator
- miqps_matpower() provides unified interface to multiple MILP/MIQP solvers.
- miqps_clex() provides a unified MILP/MIQP interface to CPLEX.
- miqps_glpk() provides a unified MILP interface to GLPK.
- miqps_gurobi() provides a unified MILP/MIQP interface to Gurobi.
- miqps_mosek() provides a unified MILP/MIQP interface to MOSEK.
- miqps_ot() provides a unified MILP interface to intlingprog().
- mosek_symbcon() defines symbolic constants for setting MOSEK options.

\section*{Other Improvements}
- Cleaned up and improved consistency of output in printpf() for generation and dispatchable load constraints.
- Modified runcpf () to gracefully handle the case when the base and target cases are identical (as opposed to getting lost in an infinite loop).
- Optional generator and dispatchable load sections in pretty-printed output now include off-line units.

\section*{Bugs Fixed}
- Fixed fatal bug in case_info() for islands with no generation.
- Fixed fatal bug in toggle_dcline() when pretty-printing results. Thanks to Deep Kiran for reporting.
- Fixed sign error on multipliers on lower bound on constraints in qps_clp() and qps_glpk().
- Fixed bug in handling of interface flow limits, where multipliers on binding interface flow limits were off by a factor of the p.u. MVA base.
- Fixed minor bug with poly2pwl(), affecting units with PMAX \(\leq 0\).
- Fixed error in qps_mosek() in printout of selected optimizer when using MOSEK 7.
- Fixed bug in hasPQcap() that resulted in ignoring generator capability curves for units whose reactive range increases as real power output increases. Thanks to Irina Boiarchuk for reporting.
- Fixed several incompatibilities with Matlab versions < 7.3.

\section*{H. 11 Version 6.0 - released Dec 16, 2016}

The Matpower 6.0 User's Manual is available online. \({ }^{69}\)

\section*{New Open Development Model}
- Matpower development has moved to GitHub! The code repository is now publicly available to clone and submit pull requests. \({ }^{70}\)
- Public issue tracker for reporting bugs, submitting patches, etc. \({ }^{71}\)
- Separate repositories for Matpower, MOST, MIPS, MP-Test, all available from https://github.com/MATPOWER/.
- New developer e-mail list (MATPOWER-DEV-L) to facilitate communication between those collaborating on MATPOWER-related development. Sign up at: http://www.pserc.cornell.edu/matpower/mailinglists.html\#devlist.

\section*{New Case Files}
- Added 9 new case files, 8 cases ranging from 1888 to 6515 buses representing the French system, and a 13,659-bus case representing parts of the of the European high voltage transmission network, stemming from the Pan European Grid Advanced Simulation and State Estimation (PEGASE) project. Thanks again to Cédric Josz and colleagues from the French Transmission System Operator. Please cite reference [43] when publishing results based on these cases.
- Added case145.m, IEEE 145 bus, 50 generator dynamic test case from the U of WA Power Systems Test Case Archive \({ }^{72}\).
- Added case33bw.m, a 33-bus radial distribution system from Baran and Wu.

\footnotetext{
\({ }^{69}\) http://www.pserc.cornell.edu/matpower/docs/MATPOWER-manual-6.0.pdf
\({ }^{70}\) https://github.com/MATPOWER/matpower
\({ }^{71}\) https://github.com/MATPOWER/matpower/issues
\({ }^{72}\) http://www.ee.washington.edu/research/pstca/dyn50/pg_tcadd50.htm
}

\section*{New Features}
- Matpower Optimal Scheduling Tool (MOST) 1.0b1 is a major new feature, implementing a full range of optimal power scheduling problems, from a simple as a deterministic, single period economic dispatch problem with no transmission constraints to as complex as a stochastic, security-constrained, combined unit-commitment and multiperiod OPF problem with locational contingency and load-following reserves, ramping costs and constraints, deferrable demands, lossy storage resources and uncertain renewable generation. See docs/MOST-manual.pdf for details.
- General mechanism for applying modifications to an existing Matpower case. See apply_changes() and idx_ct().
- Redesigned CPF callback mechanism to handle CPF events such as generator limits, nose point detection, etc. Included event \(\log\) in CPF results.
- Added options 'cpf.enforce_p_lims' and 'cpf.enforce_q_lims' to enforce generator active and reactive power limts in the continuation power flow.
- Added OPF option 'opf.use_vg' to provide a convenient way to have the OPF respect the generator voltage setpoints specified in the gen matrix.
- Experimental foundation for handling of ZIP load models in power flow (Newton, fast-decoupled only), continuation power flow, and optimal power flow (MIPS, fmincon, KNITRO, IPOPT solvers only). Currently, ZIP loads can only be specified on a system-wide basis using the experimental options 'exp.sys_wide_zip_loads.pw' and 'exp.sys_wide_zip_loads.qw'.
- Support for quadprog() under GNU Octave.
- New contributed extras:
- Plot electrically meaningful drawings of a Matpower case using plot_mpc() in extras/misc, contributed by Paul Cuffe.
- Find the maximum loadability limit of a system via an optimal power flow and dispatchable loads, using maxloadlim() in extras/maxloadlim, contributed by Camille Hamon.
- Create a quadratically-constrained quadratic programming (QCQP) representation of the AC power flow problem using using qcqp_opf() in extras/misc, contributed by Cédric Josz and colleagues.
- New functions:
- apply_changes() and idx_ct() provide a general mechanism for applying modifications to an existing Matpower case.
- feval_w_path() evaluates a function located at a specified path, outside of the Matlab path.
- mpopt2qpopt() provides a common interface for creating options struct for mi/qps_matpower() from a MATPOWER options struct.
- New function options:
- Option to call makeB(), makeBdc(), makePTDF(), scale_load(), and total_load() with full case struct (mpc) instead of individual data matrices (bus, branch, etc.).
- total_load(), which now computes voltage-dependent load values, accepts the values 'bus' and 'area' as valid values for 'load_zone' argument.

\section*{Other Improvements}
- Changed default solver order for LP, QP, MILP, MIQP problems to move Gurobi before CPLEX and BPMPD after Optimization Toolbox and GLPK.
- Added some caching to mpoption() and made minor changes to nested_struct_copy() to greatly decrease the overhead added by mpoption() when running many small problems.
- Added option 'cpf.adapt_step_damping' to control oscillations in adaptive step size control for continuation power flow.
- Added CPF user options for setting tolerances for target lambda detection and nose point detection, 'cpf.target_lam_tol' and 'cpf.nose_tol', respectively.
- Added support for Matlab Optimization Toolbox 7.5 (R2016b).
- Added support for MOSEK v8.x.
- Added tests for power flow with 'pf.enforce_q_lims' option.
- Updated network reduction code to handle cases with radially connected external buses.
- Updated versions of qcqp_opf() and qcqp_opf() in extras/misc, from Cédric Josz.
- Added "Release History" section to Appendix of manual.
- Many new tests.

\section*{Bugs Fixed}
- Fixed bug in toggle_dclines() that resulted in fatal error when used with OPF with reactive power costs. Thanks to Irina Boiarchuk.
- Fixed fatal bug in update_mupq() affecting cases where QMIN is greater than or equal to QC1MIN and QC2MIN (or QMAX is less than or equal to QC1MAX and QC2MAX) for all generators. Thanks Jose Miguel.
- Copying a field containing a struct to a non-struct field with nested_struct_copy() now overwrites rather than causing a fatal error.
- Fixed a bug in psse_convert_xfmr() where conversion of data for transformers with \(\mathrm{CZ}=3\) was done incorrectly. Thanks to Jose Marin and Yujia Zhu.
- Fixed a fatal bug in psse_convert_xfmr() affecting transformers with CW and/or CZ equal to 1. Thanks to Matthias Resch.
- Fixed a crash in have_fcn() caused by changes in OPTI Toolbox v2.15 (or possibly v2.12)
- Commented out isolated bus 10287 in case3375wp.m.
- Added code to DC OPF to return success \(=0\) for cases where the matrix is singular (e.g. islanded system without slack).
- Fixed problem in have_fcn() where SeDuMi was turning off and leaving off all warnings.
- Fixed shadow prices on variable bounds for AC OPF for fmincon, Ipopt, and KNITRO.
- In savecase() single quotes are now escaped properly in bus names.
- Generator capability curve parameters that define a zero-reactive power line no longer cause a fatal error.
- Bad bus numbers no longer cause a fatal error (after reporting the bad bus numbers) in case_info().

\section*{Incompatible Changes}
- Removed fairmax() from the public interface by moving it inside uopf(), the only place it was used.
- Removed 'cpf.user_callback_args' option and modified 'cpf.user_callback'.
- Changed name of 'cpf.error_tol' option to 'cpf.adapt_step_tol'.

\section*{H. 12 Version 7.0 - beta 1 released released Oct 31, 2018}

The Matpower 7.0b1 User's Manual is available online. \({ }^{73}\)

\section*{New Features}
- New Matpower installer script installmatpower() automatically updates Matlab or Octave paths or optionally provides the commands required to so.
- Support for additional user-defined general nonlinear constraints and costs in AC OPF.
- Support for exporting Matpower case to PSS/E RAW data format.
- Three new variants of the standard AC OPF formulation, for a total of four, including both nodal power and current balance constraints and both polar and cartesian representations of voltage. See the new opf.current_balance and opf.v_cartesian options. Thanks to Baljinnyam Sereeter.
- Three new power flow algorithms for radial distribution systems selected via the three new options for \(\mathrm{pf} . \mathrm{alg}\), namely 'PQSUM', 'ISUM', 'YSUM'. Also includes new Matpower options pf.radial.max_it and pf.radial.vcorr. See Section 4.3 on "Distribution Power Flow" for details. Thanks to Mirko Todorovski.
- Major update to OPF soft limit functionality, supporting soft limits on all AC and DC OPF inequality constraints, including branch flow constraints, bus voltage bounds, generator active and reactive bounds, branch flow and branch angle difference limits. Thanks to Eran Schweitzer.
- New options:
- pf.nr.lin_solver controls the linear solver used to compute the Newton update step in the Newton-Raphson power flow.
- pf.radial.max_it and pf.radial.vcorr are options for the new radial power flow algorithms. Thanks to Mirko Todorovski.
- cpf.enforce_flow_lims and cpf.enforce_v_lims control enforcement of branch flow and bus voltage magnitude limits in the continuation power

\footnotetext{
\({ }^{73}\) http://www.pserc.cornell.edu/matpower/docs/MATPOWER-manual-7.0b1.pdf
}
flow and cpf.flow_lims_tol and cpf.v_lims_tol control the respective detection tolerances. Thanks to Ahmad Sadiq Abubakar and Shrirang Abhyankar.
- opf.current_balance and opf.v_cartesian control formulation used for AC OPF. Thanks to Baljinnyam Sereeter.
- opf.softlims.default determines whether or not to include soft limits on constraints whose parameters are not specified explicitly in the mpc.softlims struct. For use with enhanced toggle_softlims() functionality. Thanks to Eran Schweitzer.
- opf.start replaces deprecated opf.init_from_mpc and adds a new possibility to automatically run a power flow to initialize the starting state for the OPF.
- New functions:
- calc_branch_angle calcultes voltage angle differences for branches.
- dImis_dV() evaluates the partial derivatives of nodal current balance with respect to bus voltages.
- d2Imis_dV2() evaluates the \(2^{\text {nd }}\) derivatives of nodal current balance with respect to bus voltages.
- d2Imis_dVdSg() evaluates the \(2^{\text {nd }}\) derivatives of nodal current balance with respect to bus voltages and generator injections.
- d2Abr_dV2() evaluates the \(2^{\text {nd }}\) derivatives of squared branch flows with respect to bus voltages.
- gentypes() and genfuels provide list of standard generator unit types and fuel types, respectively.
- install_matpower() installs MATPOWER by automatically modifying or, alternatively, showing needed modifications to Matlab or Octave path.
- loadshed() computes MW curtailments of dispatchable loads.
- opf_branch_ang_fcn() evaluates AC branch flow limit constraints and gradients.
- opf_branch_ang_hess() evaluates Hessian of AC branch flow limit constraints.
- opf_current_balance_fcn() evaluates AC current balance constraints and gradients.
- opf_current_balance_hess() evaluates Hessian of AC current balance constraints.
- opf_veq_fcn() evaluates voltage magnitude equality constraints and gradients.
- opf_veq_hess() evaluates Hessian of voltage magnitude equality constraints.
- opf_vlim_fcn() evaluates voltage magnitude limit constraints and gradients.
- opf_vlim_hess() evaluates Hessian of voltage magnitude limit constraints.
- opf_vref_fcn() evaluates reference voltage angle equality constraints and gradients.
- opf_vref_hess() evaluates Hessian of reference voltage angle equality constraints.
- opt_model/add_lin_constraint() to add linear constraints to an optimization model.
- opt_model/add_nln_constraint() to add nonlinear constraints to an optimization model.
- opt_model/init_indexed_name() to initialize the indices for an indexed name set of constraints, costs or variables.
- save2psse() to export a MATPOWER case to PSS/E RAW data format.
- savechgtab() to save change tables, such as those used by apply_changes, to a file.

\section*{New Case Files}
- Seven new purely synthetic cases from the ACTIVSg team (ASU, Cornell, Texas A\&M, U of Illinois, and VCU - Synthetic grids), resulting from work supported by the ARPA-E GRID DATA program. Thanks to Adam Birchfield and the ACTIVSg team.
- case_ACTIVSg200 - 200-bus Illinois synthetic model
- case_ACTIVSg500 - 500-bus South Carolina synthetic model
- case_ACTIVSg2000 - 2000-bus Texas synthetic model
- case_ACTIVSg10k - 10,000-bus US WECC synthetic model
- case_ACTIVSg25k - 25,000-bus US Northeast/Mid-Atlantic synthetic model
- case_ACTIVSg70k - 70,000-bus Eastern US synthetic model
- case_SyntheticUSA - 82,000-bus continental USA synthetic model (aggregation of case_ACTIVSg70k, case_ACTIVSg10k, and case_ACTIVSg2000, connected by 9 DC lines)

Some of these cases also include contingency tables and/or hourly load scenarios for 1 year.
- contab_ACTIVSg200
- contab_ACTIVSg500
- contab_ACTIVSg2000
- contab_ACTIVSg10k
- scenarios_ACTIVSg200
- scenarios_ACTIVSg2000
- New RTS-GMLC case from https://github.com/GridMod/RTS-GMLC.
- case4_RTS_GMLC
- Six new radial distribution system cases. Thanks to Mirko Todorovski.
- case4_dist
- case18
- case22
- case69
- case85
- case141

\section*{New Documentation}
- Two new Tech Notes, available from Matpower home page.
- Matpower Technical Note 3 "Addendum to AC Power Flows and their Derivatives using Complex Matrix Notation: Nodal Current Balance" [37]
- Matpower Technical Note 4 "AC Power Flows and their Derivatives using Complex Matrix Notation and Cartesian Coordinate Voltages" [38]
- LATEX source code for Matpower User's Manual included in docs/src, for MIPS User's Manual in mips/docs/src and for MOST User's Manual in most/docs/src.

\section*{Other Improvements}
- Update versions of included packages:
- MIPS 1.3.
- MOST 1.0.1.
- MP-Test 7.0b1.
- Continuous integration testing via GitHub and Travis-CI integration.
- Support added in core optimization model opt_model for:
- general nonlinear constraints
- general nonlinear costs
- quadratic costs
- Refactor OPF code to take advantage of new opt_model capabilities for nonlinear constraints and quadratic and nonlinear costs.
- Derivative functions now support cartesian coordinates for voltage in addition to polar coordinates.
- In the Newton power flow, for larger systems use explicit LU decomposition with AMD reordering and the 3 output argument form of \(1 u\) (to select the Gilbert-Peierls algorithm), resulting in up to a 2 x speedup in Matlab, 1.1x in Octave. Thanks to Jose Luis Marín.
- Support plotting of multiple nose curves in CPF by allowing option cpf.plot.bus to take on vector values.
- Add line for curtailed load to case_info() output.
- Change default implementation of active power line flow constraints (opf.flow_lim \(=' \mathrm{P}\) ') to use flow directly, rather than square of flow, which is now a separate option, namely opf.flow_lim = '2'. Thanks to Nico Meyer-Huebner.
- Add genfuels and gentypes to establish standard set of values for optional mpc.genfuel and mpc.gentype fields for generator fuel type and generator unit type, respectively.
- Add support for gentype and genfuel fields of Matpower case struct in extract_islands, ext2int, int2ext, load2disp and savecase.
- Add support for bus_name field of MATPOWER case struct to extract_islands, ext2int and int2ext.
- Deprecated functions:
- d2AIbr_dV2() - use dA2br_dV2() instead.
- d2ASbr_dV2() - use dA2br_dV2() instead.
- opt_model/add_constraints() - use the corresponding one of the following methods instead: add_lin_constraint(), add_nln_constraint(), or init_indexed_name().
- opt_model/add_costs() - use the corresponding one of the following methods instead: add_quad_cost(), add_nln_cost(), add_legacy_cost(), or init_indexed_name().
- opt_model/linear_constraints() - use opt_model/params_lin_constraint() instead.
- opt_model/build_cost_params() - no longer needed, incorporated into opt_model/params_legacy_cost().
- opt_model/get_cost_params() - use opt_model/params_legacy_cost() instead.

\section*{Bugs Fixed}
- Fix bug in conversion of older versions of Matpower options.
- Fix bug \#4 where some Q limits were not being respected by CPF when buses were converted to PQ by initial power flow run. Thanks to Shruti Rao.
- Fix fatal bug \#8 when calling runcpf with base and target cases with identical load and generation. Thanks to Felix.
- Fix fatal bug in get_losses when computing derivatives of reactive branch injections and fix some related tests.
- Fix \#11 fatal error encountered when running test_matpower with SDP_PF and YALMIP installed, but no SDP solver. Now checks for availability of SeDuMi, SDP3 or MOSEK before attempting to run SDP_PF tests that require solving an SDP. Thanks to Felix.
- Fix bug \#12 where the CPF could terminate early when requesting trace of the full curve with P or Q limits enforced, if a limit becomes binding at the base case. Thanks to Felix.
- Fix bug \#13 where setting all buses to type NONE (isolated) resulted in a fatal error for ext2int, runpf, runcpf and runopf. Thanks to SNPerkin.
- Fix bug \#21 where a continuation power flow that failed the first corrector step would produce a fatal error. Thanks to Elis Nycander.
- Fix bug \#23 where the continuation power flow could switch directions unexpectedly when the operating point switched from stable to unstable manifold or vice-versa after hitting a limit. Thanks to Elis Nycander and Shrirang Abhyankar.
- Fix bug \(\# 26\) where, in a continuation power flow, a reactive limit at a bus could be detected in error if multiple generators at the bus had reactive ranges of very different sizes. Thanks to Elis Nycander and Shrirang Abhyankar.
- Fix runpf handling of case where individual power flow fails during Q limit enforcement.

\section*{Incompatible Changes}
- Move included Matpower case files to new data subdirectory.
- Turning soft limits on without specifying any parameters explicitly in mpc.softlims now implements soft limits for all constraints, by default, not just branch flow limits. And the format of the input parameters in mpc.softlims has changed. See help toggle_softlims or Tables 7-9, 7-10 and 7-11 for the details.
- Swap the order of the output arguments of dSbus_dV()) for polar coordinate voltages (angle before magnitude) for consistency.
- Correct signs of phase shifter angles in Polish system cases, since they were based on the old sign convention used by Matpower prior to v3.2 (see change on \(6 / 21 / 07)\). Affects the following cases:
- case2383wp
- case2736sp
- case2737sop
- case2746wop
- case2746wp
- case3375wp

Thanks to Mikhail Khokhlov and Dr. Artjoms Obusevs for reporting.
- Remove nln.mu.l.<name> and nln.mu.u.<name> fields from OPF results struct. Use nle.lambda.<name> and nli.mu.<name> fields instead for nonlinear constraint multipliers.
- Modify order of default output arguments of opt_model/get_idx().
- Add mpopt to input args for OPF 'ext2int', 'formulation', and 'int2ext' callbacks.

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[^0]:    ${ }^{1}$ http://pserc.org/
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[^1]:    ${ }^{3}$ Versions 4.0 through 5.0 of MATPOWER were distributed under version 3.0 of the GNU General Public License (GPL) [6] with an exception added to clarify our intention to allow Matpower to interface with Matlab as well as any other Matlab code or MEX-files a user may have installed, regardless of their licensing terms. The full text of the GPL can be found at http://www.gnu. org/licenses/gpl-3.0.txt. MATPOWER versions prior to version 4 had their own license.

[^2]:    ${ }^{4}$ Matlab is available from The MathWorks, Inc. (http://www.mathworks.com/). Matpower 5 and Matpower 6 required Matlab 7 (R14), Matpower 4 required Matlab 6.5 (R13), Matpower 3.2 required Matlab 6 (R12), Matpower 3.0 required Matlab 5 and Matpower 2.0 and earlier required only Matlab 4. Matlab is a registered trademark of The MathWorks, Inc.
    ${ }^{5} \mathrm{GNU}$ Octave [4] is free software, available online at http://www.gnu.org/software/octave/. Matpower 5 and Matpower 6 required Octave 3.4, Matpower 4 required Octave 3.2.
    ${ }^{6}$ http://www.mathworks.com/support/sysreq/previous_releases.html

[^3]:    ${ }^{7}$ http://www.pserc.cornell.edu/matpower/
    ${ }^{8}$ https://github.com/MATPOWER/matpower
    ${ }^{9}$ See https://git-scm.com/downloads for information on downloading Git clients.

[^4]:    ${ }^{10} \mathrm{~A}$ good place to start is https://git-scm.com.

[^5]:    ${ }^{11}$ Do not place Matpower's files in a directory named 'matlab' or 'optim' (both caseinsensitive), as these can cause Matlab's built-in ver command to behave strangely in ways that affect Matpower.
    ${ }^{12}$ The MOST test suite is run separately by typing test_most. See the MOST User's Manual for details.

[^6]:    ${ }^{13}$ This describes version 2 of the Matpower case format, which is used internally and is the default. The version 1 format, now deprecated, but still accessible via the loadcase and savecase functions, defines the data matrices as individual variables rather than fields of a struct, and some do not include all of the columns defined in version 2.

